MathWorks 12

Workbook

Pacific Educational Press

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How to Use This Book

This workbook is a companion to *MathWorks 12* student resource, the authorized resource for the Western and Northern Canadian Protocol (WNCP) course, Apprenticeship and Workplace Mathematics. The *MathWorks 12 Workbook* is a valuable learning tool when used in conjunction with the student resource, or on its own. It emphasizes mathematical skill-building through worked examples and practice problems.

Here, you will learn and use the practical mathematics required in the workplace. Whether you plan to enroll in college, learn a trade, or enter the workforce after graduating from secondary school, the mathematical skills in this workbook will support you at work and in your daily life.

The *MathWorks 12 Workbook* contains eight chapters. Chapters are divided into sections, each focusing on a key mathematical concept. Chapters include the following features.

Review

Most chapters open with a review of mathematical processes and terms you will need to understand to complete the chapter's lessons. Practice questions are included.

Example

Each example includes a problem and its solution. The problem is solved step by step. Written descriptions of each mathematical operation used to solve the problem are included.

New Skills

The chapter's core mathematical concepts are introduced here. This section often includes real-world examples of where and how the concepts are used.

Build Your Skills

This is a set of mathematical problems for you to solve. It appears after each new concept is introduced and will let you practise the concepts you have just learned about. Build Your Skills questions often focus on workplace applications of mathematical concepts.

Practise Your New Skills

The section's key concepts are presented as review problems at the end of each chapter section.

Chapter Test

At the end of each chapter, a chapter test is provided for review and assessment of learning.

Definitions

New mathematical terms are defined in the sidebar columns. They are also included in the glossary at the back of the book.

Glossary

Definitions for mathematical terms are provided here. To increase understanding, some glossary definitions include illustrations.

Appendix

Contains conversion tables, formulas, graph paper, and other useful resources.

Chapter 1

Linear Relations

Car manufacturers specify the fuel consumption of their vehicles to help you understand how much fuel the vehicles require. Fuel consumption is often specified in miles per gallon (mpg). This fuel consumption rate indicates a linear relation—a car will be able to be driven approximately the same number of miles per gallon of fuel used.

Linear Relations in Tables and Graphs 1.1

REVIEW: WORKING WITH SLOPE

Slope refers to the steepness of a line. Other words are sometimes used to refer to slope. When talking about a roof, it can be called the pitch. On a road, can be called the grade. Slope is a ratio of the change in the vertical distance over the change in the horizontal distance. This can be written as:

$$\text{slope} = \frac{\Delta \text{ vertical distance}}{\Delta \text{ horizontal distance}} \text{ or } m = \frac{\Delta y}{\Delta x}$$

The variable m is used to represent slope. The change in vertical distance is also called the rise, and the change in horizontal distance is also called the run. Slope can therefore be expressed as:

$$m = \frac{\text{rise}}{\text{run}}$$

slope: in a linear relation, the ratio of the vertical distance over the horizontal distance; the formula is: $m = \frac{\text{rise}}{\text{run}}$

$m = \frac{\Delta y}{\Delta x}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

The symbol Δ is the Greek letter delta. It means "change" or "difference."

Example 1

a) Calculate the slope of a line that has a rise of 15 m and a run of 6 m.

b) If the same line is extended to a run of 9 m, what will be its total rise?

SOLUTION

a) Use the formula for slope.

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{15}{6}$$

$$m = \frac{15 \div 3}{6 \div 3}$$

$$m = \frac{5}{2} \text{ or } 2.5$$

The slope can be expressed as a fraction or as a decimal. It is $\frac{5}{2}$ or 2.5.

Slope does not have units because it is a ratio, not a measurement.

b) Use the slope calculated in a). Solve for the rise.

$$m = \frac{\text{rise}}{\text{run}}$$

$$2.5 = \frac{\text{rise}}{9}$$

$$9 \times 2.5 = \frac{\text{rise}}{9} \times 9$$

$$9 \times 2.5 = \text{rise}$$

$$22.5 \text{ m} = \text{rise}$$

The total rise will be 22.5 m.

Building codes are set by governments to ensure that structures in their jurisdiction are safely built. Building codes specify the slope at which wheelchair ramps can be built.

BUILD YOUR SKILLS

1. What is the slope of a wheelchair ramp that has a rise of 3 feet and a run of 21 feet?

2. The slope of a staircase is 0.95. If it rises 250 cm, what is the run?

3. Marina is installing pipes. The pipes must drop 4 cm for every 1.75 m of horizontal distance. How much will the pipes drop in a horizontal distance of 20 m?

NEW SKILLS: WORKING WITH LINEAR RELATIONS

A graph on a coordinate plane shows the relationship between the independent variable and the dependent variable. A **linear relation** is a relationship that forms a straight line when the points are plotted on the graph. The slope of a straight line is constant. If a line passes through the **origin** (0, 0), it is called a **direct linear relation**.

For more details, see page 13 of *MathWorks 12*.

linear relation: a relationship between two variables that forms a straight line when you plot values of ordered pairs on a graph

origin: the intersection of the *x*- and *y*-axes of a coordinate plane; identified by the coordinates (0, 0)

direct linear relation: a linear relationship that includes the ordered pair (0, 0)

independent variable: a variable whose value may be freely chosen; represented as the *x*-value in an ordered pair

dependent variable: a variable whose value relies on the values of the other variables; represented as the *y*-value in an ordered pair

Example 2

Kalinda babysits on weekends and charges $12.50/hour. The following table of values shows her earnings based on the number of hours she babysits.

KALINDA'S EARNINGS FROM BABYSITTING

Hours of work	*Total earnings*
0	0
1	$12.50
2	$25.00
3	$37.50
4	$50.00
5	$62.50
6	$75.00
7	$87.50

a) Which is the **independent variable** and which is the **dependent variable**?

b) Use the data to draw a graph. Should you connect the points on the graph? Why or why not?

c) List three characteristics that you observe about the shape of the graphed data.

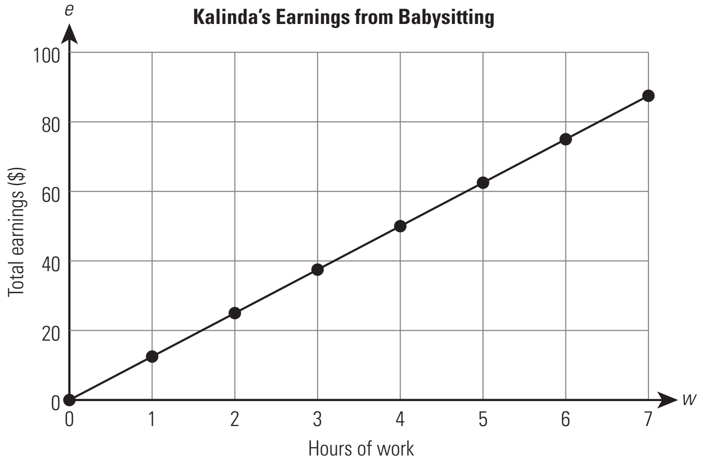
d) What is the slope of the graph? What does it represent?

e) How long will it take Kalinda to earn $150.00?

SOLUTION

a) The independent variable is the time, or hours of work. It will go on the x-axis. The earnings depend on the amount of time worked, so earnings is the dependent variable. It will go on the y-axis.

b)



Even though the data is not completely continuous because Kalinda likely earns part of her hourly wage for partial hours of work, you can connect the points on the graph. The points are joined to allow you to see the trend in the data, even though she is likely only paid to the nearest quarter-hour.

c) The shape of the graphed data shows:

- a straight line;
- a line that passes through the point (0, 0); and
- a line that slopes upward to the right.

d) To find the slope, start by choosing any two points on the graph, such as (0, 0) and (4, 50). Slope is calculated by dividing the change in vertical distance by the change in horizontal distance. You can calculate this using the data points you chose.

$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute the y value of the second data point (50) in for y_1. Substitute the y value of the first data point (0) in for y_2. Do the same for the x values.

$$m = \frac{50 - 0}{4 - 0}$$

$$m = \frac{50}{4}$$

$$m = 12.5$$

The slope is 12.5. It represents Kalinda's earnings per hour ($12.50/hour).

ALTERNATIVE SOLUTION

When you are calculating the slope of a direct linear relationship, which passes through (0, 0), you only need to pick one data point on the line to calculate the slope, such as (1, 12.50).

$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{12.50}{1}$$

$$m = 12.5$$

e) Kalinda earns $12.50 per hour. Divide $150.00 by her hourly rate.

$150.00 ÷ $12.50/hour = 12 hours

Kalinda will have to babysit for 12 hours to earn $150.00.

BUILD YOUR SKILLS

4. Émilie wears a pedometer to count the number of steps she walks to work every day. The following table shows the number of steps she walks on average, and the amount of time spent walking.

   a) Draw a graph of the data. Should you connect the points on the graph? Why or why not?

ÉMILIE'S WALKING DISTANCE	
Time (minutes)	*Number of steps*
0	0
10	820
15	1230
35	2870

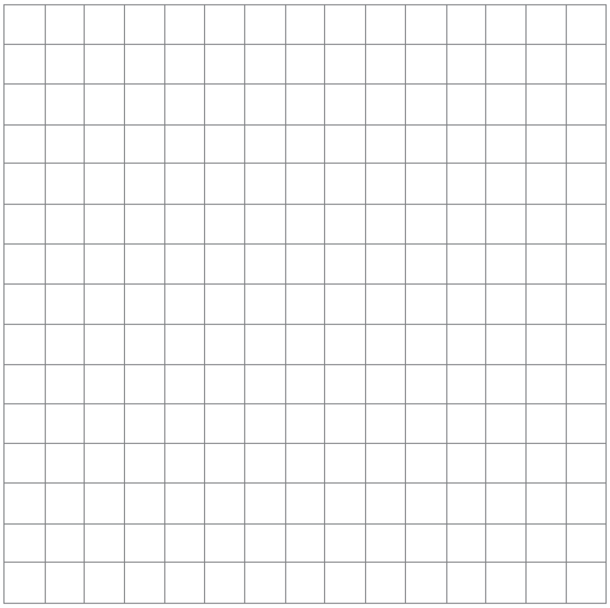
On a graph with points A, B, and C, if the slope between points A and B is the same as between points B and C, then points A, B, and C are on a line.

   b) Calculate the slope. Does the graph appear to be linear?

   c) What does this means about Émilie's speed or number of steps?

5. A water tank is being filled with water at a rate of 3.5 gallons per minute.

   a) Make a table of values that shows the amount of water in the tank each minute, from 0 to 5 minutes. Draw a graph of the data.



   b) What is the slope of the line? What does the slope tell you?

   c) If the tank holds 25 gallons of water, approximately how long will it take to fill it?

Organic fertilizers are made of naturally occuring materials such as kelp, compost, and manure. They improve the soil's long-term productivity, while providing plants with the nutrients they need.

6. A type of dry fertilizer must be dissolved in water before being applied to plants. The fertilizer container shows the following mixing guidelines.

FERTILIZER MIXING GUIDELINES	
Amount of water (US gal)	*Amount of fertilizer (oz)*
1	0.42
5	2.1
10	4.2

a) Draw a graph of the data.

b) Does the graph show a direct linear relation? Explain your reasoning.

c) How much water is needed for 2.94 oz of fertilizer?

NEW SKILLS: WORKING WITH PARTIAL VARIATION

In the above situations, the graphs all passed through (0, 0) and were called direct linear relations. Not all lines pass through the origin. If a straight line does not pass through (0, 0), it is called a **partial linear relation**. The point at which the graph crosses the y-axis is called the y-intercept.

partial linear relation: a linear relation that does not include the ordered pair (0, 0)

For more details, see page 17 of *MathWorks 12*.

Example 3

Many tradespeople run their own small businesses. In order to properly manage his electrical business, Loris needs to set payment rates and bill his clients accordingly.

Loris is an electrician. When he makes house calls, he charges $45.00 plus $35.00 for each hour of work.

a) Create a table of values showing Loris's earnings after each hour of work, up to 5 hours.

b) Draw a graph of the data. Is this a linear relation? Why or why not? If it is linear, is it partial or direct?

c) What is the slope of the graph? What does it represent?

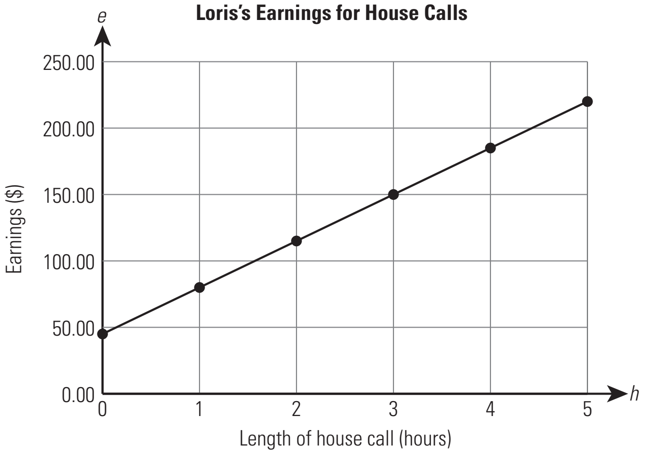
d) What is the y-intercept? What does it represent?

SOLUTION

a)

LORIS'S EARNINGS FOR HOUSE CALLS	
Length of the house call (hours)	*Earnings*
0	$45.00
1	$80.00
2	$115.00
3	$150.00
4	$185.00
5	$220.00

b)



Yes, this is a partial linear relation. The graph forms a straight line. Loris's earnings increase proportionally with the length of the house call.

c) Choose two points on the graph, such as (0, 45) and (5, 220).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{220 - 45}{5 - 0}$$

$$m = \frac{175}{5}$$

$$m = 35$$

The slope is 35. It represents Loris's hourly charge of $35.00 that is charged in addition to the base fee of $45.00.

d) Examine the graph and find what the y value is when x equals 0. When x is 0, y is 45. The y-intercept is 45. This represents Loris's base fee of $45.00 for each house call.

BUILD YOUR SKILLS

7. The table below shows the rates charged by a taxi company for different distances driven.

Remember to label your graph.

a) Draw a graph of the data.

TAXI CHARGES	
Distance driven (km)	*Cost*
0	$4.50
4.2	$10.80
6.8	$14.70
12.0	$23.50

b) Choose three points on the line. Calculate the slope between the first two points, then between the second and third points. What do your calculations tell you about the relationship between distance driven and cost?

c) What does the slope represent?

d) What does the *y*-intercept of the graph represent?

8. Kai is a salesperson at a home appliance store. He earns a base salary of $440.00 per week, plus a commission of 5% on all his sales.

a) Create a table of values that shows Kai's weekly earnings if he makes sales of $500.00, $1000.00, $2000.00, $2500.00, or $3000.00 in one week. Draw a graph of the data.

KAI'S EARNINGS PER WEEK	
Sales	*Earnings*
$500.00	
$1000.00	
$2000.00	
$2500.00	
$3000.00	

b) Describe the graph. Is the relation linear? Does the graph pass through the origin?

c) Use the graph to extrapolate how much Kai would earn if he made sales of $4000.00 in one week.

d) What is the slope of the line? What does it represent?

NEW SKILLS: WORKING WITH NEGATIVE SLOPE

A car's fuel consumption depends on driving speed. The stop-and-start driving in a traffic jam requires more fuel that driving at a constant speed on a highway.

In the graphs you have worked with so far in this chapter, the line always went up from left to right. That is, the graphs all had a positive slope. If a graph, either for a direct linear relation or for a partial linear relation, goes down from left to right, it has a negative slope.

Example 4

Josie's car has a full tank of gas, 40 litres. She knows that if she travels at a fairly constant speed, her car uses about 1 litre of fuel for every 9 kilometres of driving.

a) Make a table of values to represent the amount of fuel in Josie's car after she has driven 0 km, 45 km, 90 km, and 135 km.

b) Draw a graph of the data. Should you connect the points on the graph? Why or why not?

c) Calculate the slope of the line. What does it represent?

d) Explain why the slope is negative.

e) About how far can Josie travel before running out of fuel?

SOLUTION

a) Josie uses 1 L of fuel for every 9 km of driving. Calculate how much fuel Josie needs to drive 45 km.

$$45\text{ km} \times \frac{1\text{ L}}{9\text{ km}} = 5\text{ L}$$

She will need 5 L to drive 45 km.

$$40 - 5 = 35\text{ L}$$

There will be 35 L of fuel remaining.

After 90 km:

$$90\text{ km} \times \frac{1\text{ L}}{9\text{ km}} = 10\text{ L}$$

$40 - 10 = 30$ L

There will be 30 L remaining.

After 135 km:

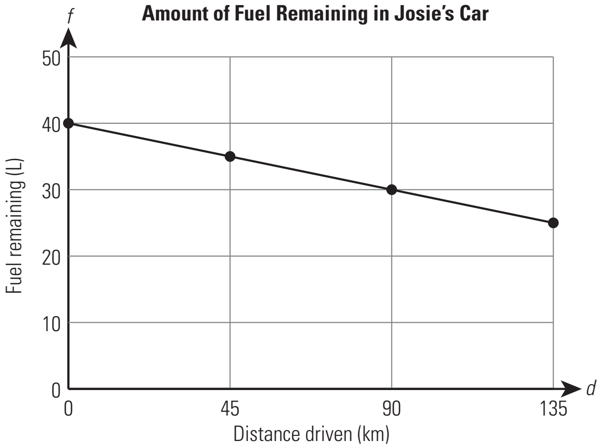
$$135\text{ km} \times \frac{1\text{ L}}{9\text{ km}} = 15\text{ L}$$

$$40 - 15 = 25\text{ L}$$

There will be 25 L remaining.

AMOUNT OF FUEL REMAINING IN JOSIE'S CAR	
Distance driven (km)	*Fuel remaining (L)*
0	40
45	35
90	30
135	25

b)



The points on the graph can be connected, because the data is continuous. The amount of fuel remaining decreases proportionally as the distance driven increases.

c) Choose two points on the graph, such as (0, 40) and (90, 30).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{30 - 40}{90 - 0}$$

$$m = \frac{-10}{90}$$

$$m = -\frac{1}{9}$$

The slope is $-\frac{1}{9}$ or about –0.11. It represents the amount of fuel used per kilometre of driving.

d) The slope is negative because the amount of fuel in the tank is decreasing as the distance driven increases.

e) Extrapolate by extending the line to reach the x-axis. It meets the axis at about 370 km, so Josie can go travel about 370 km before running out of fuel.

ALTERNATIVE SOLUTION

Use proportional reasoning. Let x represent the distance Josie can drive on a full tank. She travels 9 km using 1 L of fuel, and there are 40 L of fuel in her car.

$$\frac{9 \text{ km}}{1 \text{ L}} = \frac{x \text{ km}}{40 \text{ L}}$$

$$\frac{9}{1} = \frac{x}{40}$$

$$40 \times \frac{9}{1} = \frac{x}{40} \times 40$$

$$40 \times 9 = x$$

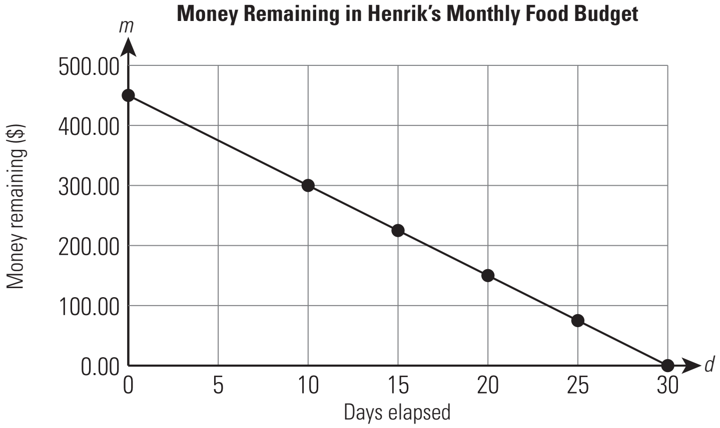
$$360 \text{ L} = x$$

Josie can travel about 360 km before running out of fuel.

Note: Using proportional reasoning is usually more accurate than extrapolation, because the graph is too small and the drawn line is too thick to calculate the exact value.

BUILD YOUR SKILLS

9. Henrik has a monthly food budget of $450.00. The following graph shows an approximation of how much money he has left in his budget each day.



Buying local produce ensures the freshest food, stimulates the local economy, and builds relationships between local growers and the community.

a) Estimate how much money he has left in his food budget after 15 days.

b) What is the slope of the line? What does it represent?

10. A 500-litre tank of water is leaking at a rate of 10 litres every three hours.

    a) Create a table of values that shows the amount of water remaining after 3 h, 9 h, 15 h, and 24 h, and draw a graph of the data.

    b) What is the slope of the line?

    c) About how long will it take for the tank to empty?

NEW SKILLS: IDENTIFYING NON–LINEAR RELATIONS

Not all data form a straight line when graphed. Non-linear graphs can take many different forms.

Example 5

A second-hand store has a policy that if an item has not sold, it drops in price by 30% every two weeks. A wool coat started out at $50.00.

a) Create a table of values showing the price of the coat after 2, 4, 6, and 8 weeks.

b) Draw a graph of the data.

c) Is the graph linear or non-linear? Use slope to support your answer.

SOLUTION

a) Calculate the price of the coat after 2 weeks. The price decreases by 30%, so the price will be 70% of the original.

After 2 weeks: $50.00 × 0.70 = $35.00

After 4 weeks: $35.00 × 0.70 = $24.50

After 6 weeks: $24.50 × 0.70 = $17.15

After 8 weeks: $17.15 × 0.70 = $12.01

PRICE OF SECOND-HAND COAT, BY TIME ELAPSED

Week	*Price*
0	$50.00
2	$35.00
4	$24.50
6	$17.15
8	$12.01

b)

c) No, the graph does not appear to be linear. The change in the x value is constant, but the changes in y decrease each time. Choose two pairs of points,

such as (0, 50) and (4, 24.50), and (2, 35) and (6, 17.15). Find the slope between the pairs of points.

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} \qquad m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{24.50 - 50}{4 - 0} \qquad m_2 = \frac{17.15 - 35}{6 - 2}$$

$$m_1 = \frac{-25.5}{4} \qquad m_2 = \frac{-17.85}{4}$$

$$m_1 = -6.375 \qquad m_2 = -4.4625$$

The slopes between the two pairs of points are not the same, so the graph is not linear.

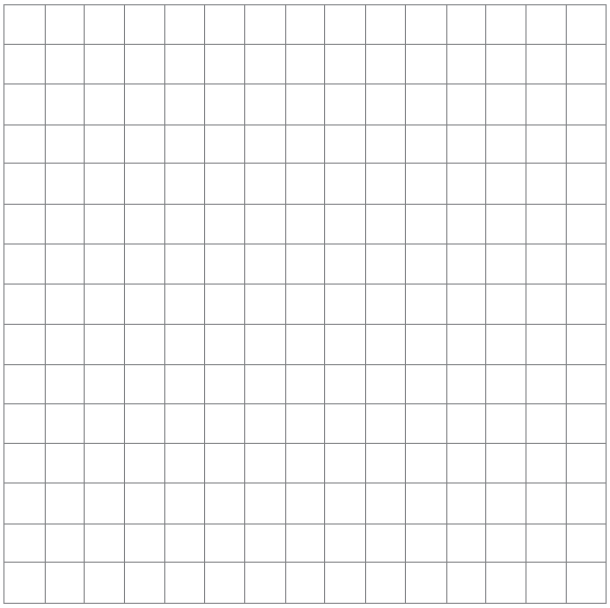
BUILD YOUR SKILLS

Many cafés specialize in latte art, with baristas able to draw complex designs in the milk as it is poured into a coffee. Baristas even compete in latte art during the annual Canadian Barista Championship.

11. Ampreet owns a café. One popular breakfast menu item is bottled orange juice. Ampreet tracks sales to find out how many bottles he should order per week. The table below indicates the total number of bottles sold by the end of each day.

BOTTLES OF ORANGE JUICE SOLD	
Day	*Number of bottles sold*
1	56
2	108
3	162
4	215
5	269
6	321

a) Draw a graph to represent the data.



b) Does the graph appear to be linear? Calculate the slope for two sections of the graph to check your answer.

12. The area of a circle is calculated using the following formula.

$$A = \pi r^2$$

a) Complete the table of values below, showing the radius and corresponding area of circles of various sizes.

AREA OF CIRCLE COMPARED TO RADIUS	
Radius (cm)	*Area (cm²)*
2	
3	
8	
12	

b) Draw a graph of the data. Is the relationship linear? Explain your answer.

PRACTISE YOUR NEW SKILLS

1. A ramp has a rise of 1.5 m for every 12 m of run.

   a) What is the slope of the ramp?

   b) What will be the total rise of the ramp when it reaches a run of 90 m?

2. Patrice hires a plumber to repair a burst pipe in his bathroom. The plumber charges a flat rate to make the house call, and then charges an additional hourly fee.

   a) The following table shows the cost of plumbing work, according to the number of hours required. Draw a graph of the data. Is this a linear relation? Why or why not?

COST OF PLUMBING WORK	
Hours of work	*Price*
1	\$51.00
2	\$74.50
3	\$98.00
4	\$121.50
5	\$145.00

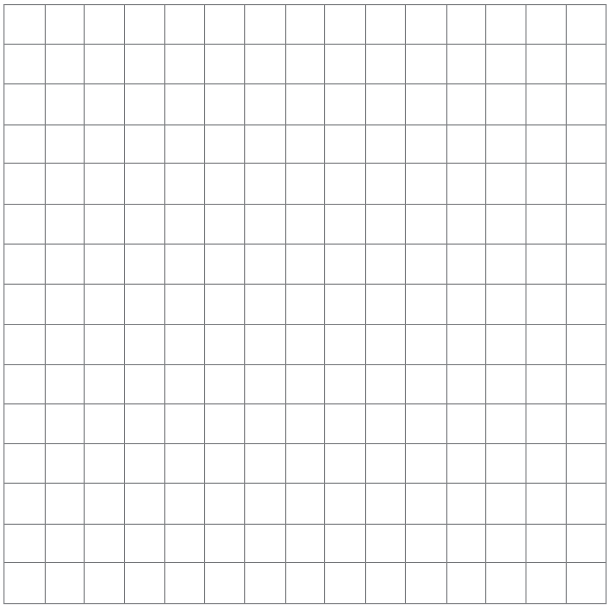
   b) What is the slope of the graph? What does it represent?

c) Extend the line to the y-axis. What is the y-intercept of the graph, and what does it represent?

3. Jeremy earn \$18.50 per hour as a tutor.

a) Create a table of values indicating his earnings per hour, from 1 to 8 hours of tutoring.

b) Draw a graph of the data. Should the points on the graph be connected? Why or why not?



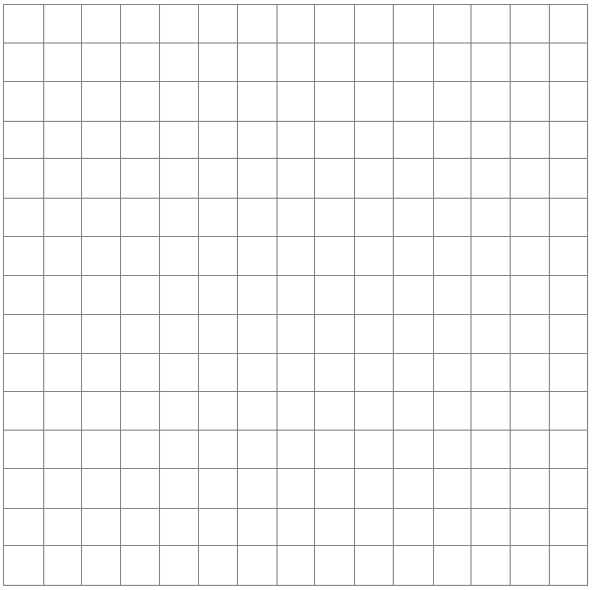
4. Ama earns \$450.00 per week, and she puts 20% of her earnings into a savings account each week.

   a) Create a table of values to represent how much she will have in the bank at the end of each week for 5 weeks. Draw a graph of this data.

   b) What is the slope of the graph? What does it represent?

5. Li Wei talked to a real estate agent who said that in her town, in 2007, the average house price was $135 000.00. Since then, it has risen an average of 2% per year.

   Create a table of values to indicate the house prices from 2007 to 2012, and use it to create a graph. Is the graph linear or non-linear? Use slope to support your answer.



1.2 The Equation of a Line

NEW SKILLS: WORKING WITH EQUATIONS OF DIRECT LINEAR RELATIONS

In section 1.1, you saw that the slope of a line can be calculated as follows.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

For a direct linear relation, which passes through the point (0, 0), this can be simplified to:

$$m = \frac{y_2}{x_2}$$

If you know the slope of the line, you can choose any x-value and calculate the corresponding y-value by substituting in the known values. If you reorder the equation to solve for y, you get the following equation:

$$y = mx$$

This equation can be used for any direct linear relation to calculate:

- the slope if one point (x, y) on the graph is known;
- the y-value for a given x-value, if the slope is known; or
- the x-value for a given y-value, if the slope is known.

For more details, see Linear 27–28 of *MathWorks 12*.

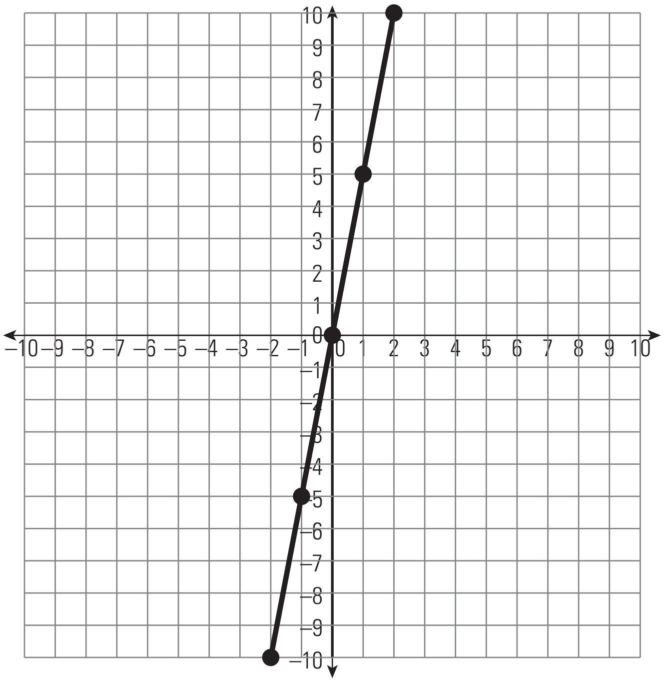
Example 1

For the following equation, calculate the values of y for the integer values of x from –2 to 2. Draw a graph of the equation:

$$y = 5x$$

SOLUTION

y* = 5*x	
x	*y*
–2	5(–2) = –10
–1	5(–1) = –5
0	5(0) = 0
1	5(1) = 5
2	5(2) = 2

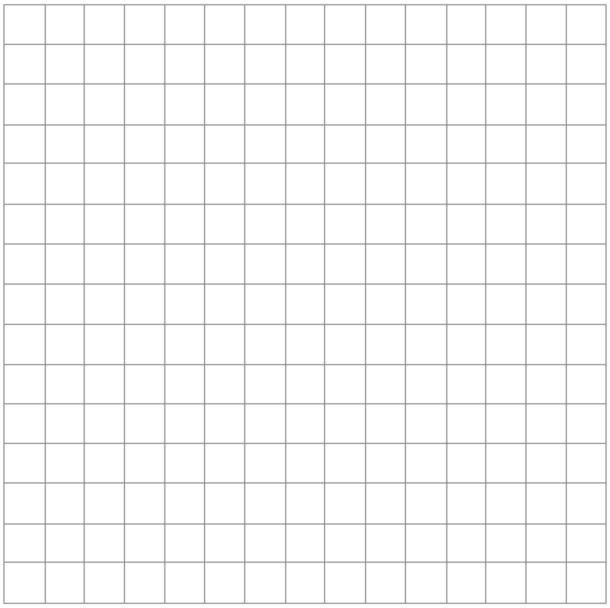


BUILD YOUR SKILLS

1. For each of the following equations, calculate the values of *y* for the values of *x* given in the table. Draw a graph of both of the equations on the same coordinate plane. What similarities and differences do you notice with respect to slope?

y* = –4*x	
x	*y*
–2	
–1	
0	
1	
2	

y* = 4*x	
x	*y*
–2	
–1	
0	
1	
2	



NEW SKILLS: WRITING EQUATIONS FOR DIRECT LINEAR RELATIONS

The variables y and x are just placeholders—they can be replaced with any letters.

When you know that the relationship between two variables is direct and linear, you can write an equation between them using the equation:

$$y = mx$$

For more details, see page 29 of *MathWorks 12*.

Example 2

Katrina earns $12.75 per hour.

a) Make a table of values showing Katrina's earnings for 0 to 5 hours, and draw a graph of the data.

b) What is the slope of the equation? What does it represent?

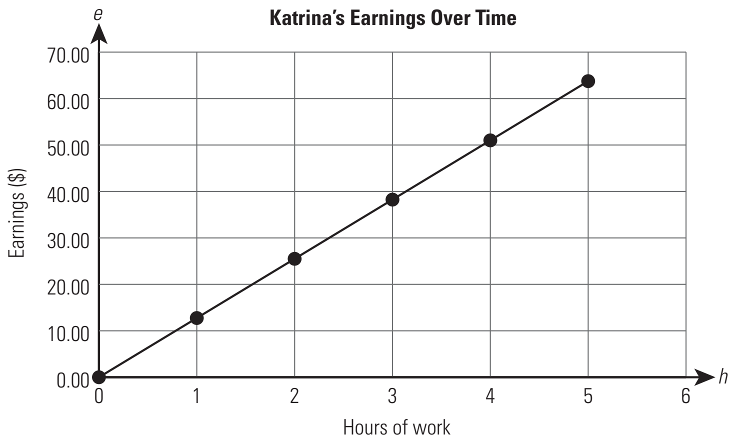
c) Write an equation that describes the relationship between Katrina's hours of work and her total earnings.

d) Use the equation to calculate how much money Katrina will earn in 5.5 hours.

SOLUTION

a)

KATRINA'S EARNINGS OVER TIME	
Hours of work	*Earnings*
0	0
1	$12.75
2	$25.50
3	$38.25
4	$51.00
5	$63.75



b) The graph shows a direct linear relation, because the line passes through the origin in a straight line. Calculate the slope.

$m = \frac{y}{x}$

$m = \frac{12.75}{1}$

$m = 12.75$

The slope represents Katrina's earnings per hour.

c) The relation can be expressed in the form:

$y = mx$

Substitute in the names of the variables.

Katrina's total earnings = 12.75 × (hours worked)

d) Katrina's total earnings = 12.75 × (hours worked)

Katrina's total earnings = 12.75 × 5.5

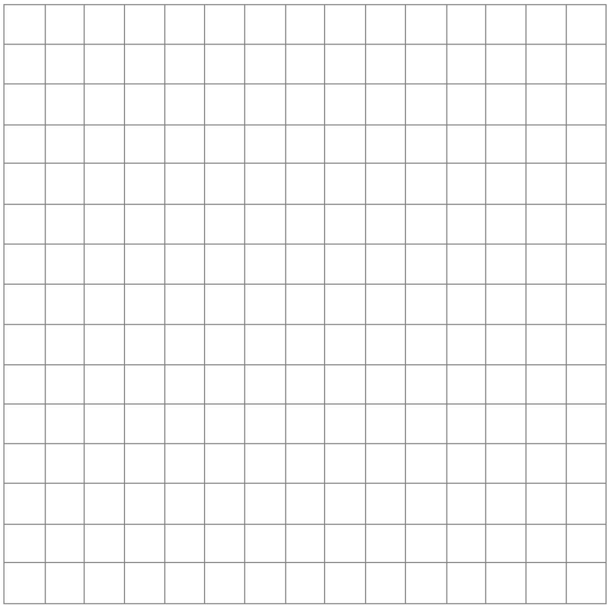
Katrina's total earnings ≈ \$70.13

In 5.5 hours of work, Katrina will earn \$70.13.

BUILD YOUR SKILLS

2. One inch is equal to 2.54 centimetres.

a) Draw a graph that shows the relationship.



b) What is the slope of the graph?

c) Write an equation that describes the relationship between inches and centimetres.

d) Use the equation to calculate how many centimetres are equal to 18 inches.

3. One kilogram is equal to 2.2 pounds.

a) Write an equation that can be used to change kilograms to pounds.

b) How many pounds are equivalent to 15 kg?

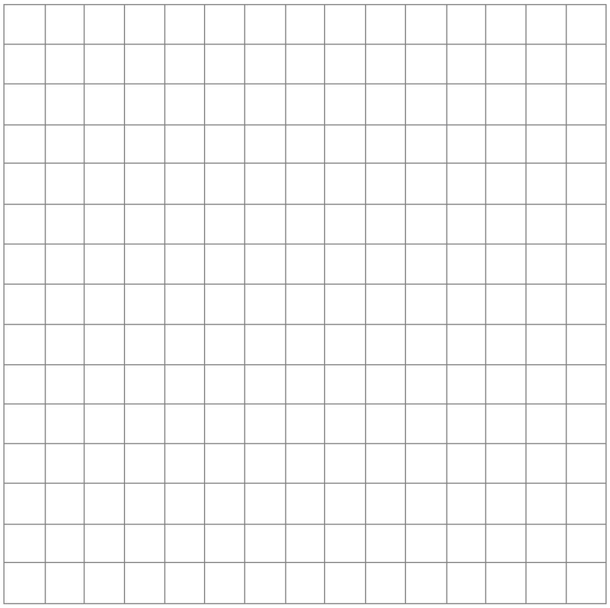
Appliance makers are always looking for ways to build models that are quieter and more energy efficient. Salespeople need to know about the differences between models in order to meet the needs of their customers.

4. Iqbal works as a salesperson in an appliance store. His income is a 20% commission on his sales.

a) Fill in the table of values showing Iqbal's earnings for the following sales amounts.

IQBAL'S EARNINGS BASED ON SALES	
Total sales	*Earnings*
0	
$1000.00	
$2500.00	
$5000.00	

b) Draw a graph of the data.



c) Write an equation that describes the relationship between Iqbal's sales and his income.

Example 3

Damien works at a paint supply store. Most customers buy paint cans in large quantities, so Damien has made the following table as a quick reference for the cost of different quantities of cans of a particular brand of paint.

TOTAL COST OF PAINT	
Number of cans	*Cost*
2	\$49.00
4	\$98.00
8	\$196.00
14	\$343.00

a) If the data were drawn on a graph, what would be the slope?

b) Write an equation that represents the relationship between total cost and number of paint cans purchased.

c) How many paint cans can be purchased for \$245.00?

SOLUTION

a) The data represents a direct linear relation, because each can of paint costs the same amount and 0 cans cost \$0.

If the data were graphed, the number of cans would be the x-value (the independent variable) and the cost would be the y-value (the dependent variable). Therefore, the line would include the following points: (0, 0), (2, 49.00), (4, 98.00), (8, 196.00), and (14, 343.00).

Calculate the slope using one of these points.

$$m = \frac{y}{x}$$

$$m = \frac{98.00}{4}$$

$$m = 24.5$$

The slope is 24.5. It represents the cost per can of paint (\$24.50).

b) The equation will have the form:

$y = mx$

Substitute in the names of the variables and the slope.

total cost = \$24.50 × (number of paint cans)

c) Substitute the total cost into the formula and solve for the number of paint cans.

$$\text{total cost} = \$24.50 \times (\text{number of paint cans})$$

$$\$245.00 = \$24.50 \times (\text{number of paint cans})$$

$$\frac{\$245.00}{\$24.50} = \frac{\$24.50 \times (\text{number of paint cans})}{\$24.50}$$

$$\frac{\$245.00}{\$24.50} = \text{number of paint cans}$$

$$10 = \text{number of paint cans}$$

For \$245.00, you could buy 10 paint cans.

BUILD YOUR SKILLS

5. The following table shows the depth of water in a water tank that is slowly being filled. The tank is being filled at a constant rate.

DEPTH OF WATER, BY TIME ELAPSED	
Time elapsed (h)	*Depth of water (cm)*
2	24
5	60
7	84
12	144

a) If the data were drawn on a graph, what would be the slope? What does the slope represent?

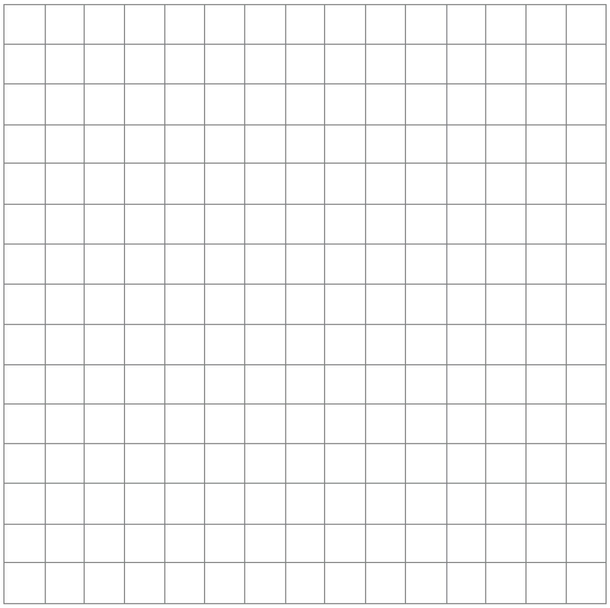
b) Write an equation that represents the relationship between the time elapsed and the depth of water.

c) Calculate how long it would take to fill the tank if it is 360 cm deep.

6. Karen worked as a gardener during the summer. The following table shows her earnings for four days, based on the number of hours she worked.

KAREN'S EARNINGS, BY HOURS OF WORK	
Hours of work	*Earnings*
3	$27.00
4	$36.00
5	$45.00
8	$72.00

a) Draw a graph of the data.



b) Does the graph appear to be linear? How do you know?

c) How much does Karen earn per hour?

d) Write an equation that shows the relationship between Karen's hours of work and her earnings.

e) How many hours does Karen need to work to earn $99.00?

NEW SKILLS: WORKING WITH EQUATIONS OF PARTIAL LINEAR RELATIONS

In a partial linear relation, the graph forms a straight line, but it does not pass through the origin. The point at which the line crosses the y-axis is called the **y-intercept**. The y-intercept needs to be taken into account in the equation of the line.

y-intercept: the y-value of the point where the graph of a relation intersects the y-axis

A partial linear relation can be represented as the following equation, where b is the y-intercept.

$$y = mx + b$$

For more information, see page 33 of *MathWorks 12*.

Example 4

a) Write an equation for a line with a slope of 5 and a y-intercept of 6. What is the value of y when x is 5?

b) Write an equation for a line with a slope of –2 and a y-intercept of –3. What is the value of x when y is –9?

SOLUTION

a) Substitute the given values into the equation for a partial linear relation.

$y = mx + b$
$y = 5x + 6$

Solve for y when x equals 5.

$y = 5x + 6$
$y = 5(5) + 6$
$y = 25 + 6$
$y = 31$

b) Substitute the given values into the equation for a partial linear relation.

$y = mx + b$
$y = -2x - 3$

Solve for x when y equals –9.

$$\begin{aligned} y &= -2x - 3 \\ -9 &= -2x - 3 \\ 3 - 9 &= -2x \\ -6 &= -2x \\ \frac{-6}{-2} &= \frac{-2x}{-2} \\ 3 &= x \end{aligned}$$

BUILD YOUR SKILLS

7. Write the equation of a line with:

   a) a slope of 6 and a y-intercept of –5

   b) a slope of $\frac{3}{4}$ and a y-intercept of 15

8. Write the equation of a line with a slope of –5 and a y-intercept of –3. What is the value of y when x is 3?

9. a) Write the equation of a line with a slope of 12.5 and a y-intercept of 25.5.

b) What is the value of y when x is 10?

c) What is the value of x when y is 213?

Example 5

Vikram is planning a banquet dinner. The caterer charges a flat fee of $150.00 plus $420.00 for every 10 people attending.

a) Make a table of values to represent the cost of 10, 20, 30, 40, or 50 people attending the banquet.

b) Draw a graph of the data.

c) What is the slope of the graph? What does it represent?

d) What is the y-intercept? What does it represent?

e) Write an equation that represents the relationship between the cost of the banquet and the number of people attending.

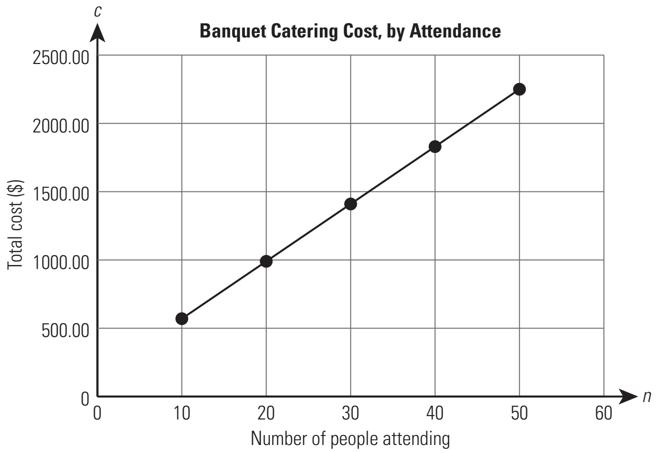
SOLUTION

a)

BANQUET CATERING COST, BY ATTENDANCE

Number of people attending	*Total cost*
10	$570.00
20	$990.00
30	$1410.00
40	$1830.00
50	$2250.00

b)



c) Choose two points on the line, such as (10, 570) and (20, 990). Calculate the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{990 - 570}{20 - 10}$$

$$m = \frac{420}{10}$$

$$m = 42$$

The slope represents the cost per person that is charged on top of the $150.00 flat fee.

d) Calculate the y-intercept by substituting one of the known coordinates, such as (10, 570) into the equation of the line.

$$y = mx + b$$

$$570 = (42)(10) + b$$

$$570 = 420 + b$$

$$570 - 420 = b$$

$$150 = b$$

The y-intercept is 150. It represents the flat fee charged by the caterer.

e) Substitute the slope and y-intercept into the equation of the line.

$y = mx + b$

total cost = 42 × (number of people) + 150

BUILD YOUR SKILLS

10. Get-Fit Fitness Centre charges $75.00 to join, then $25.00 per month to be a member.

    a) Draw a graph that shows the cost of being a member for 1, 5, and 10 months.

    b) Write an equation that shows the relationship between the number of months of membership and the total cost.

    c) How much would it cost to be a member for 9 months?

Many fitness centres offer a variety of fitness classes to appeal to those wanting more variety in their workouts. Aerial yoga combines traditional yoga postures with aerial acrobatics and pilates. A suspended hammock is used to perform the hybrid postures.

11. The temperature on a winter's evening decreases by approximately 1.5°C every hour from 4:00 pm as the evening progresses.

    a) If the temperature was 3°C at 4:00 pm, draw a graph to represent the decline in temperature until 10:00 pm.

    b) What is the slope of the graph? What does it represent?

    c) What would the temperature be at 10:00 pm?

    d) Do you think this trend would continue all night? Why or why not?

12. Ellie is a heating, ventilation, and air conditioning (HVAC) repair person. When she makes house calls, she charges a flat fee of $50.00 plus $30.00 per hour.

    a) Write an equation showing the relationship between the number of hours of work and the total cost.

    b) How many hours would Ellie have had to work to charge her customer $230.00?

Example 6

Charlie wants to add a long-distance plan to his cell phone plan. He is comparing two options:

- Option 1 is a rate of $0.25/minute
- Option 2 charges a flat fee of $5.00/month, plus a fee of $0.10/minute.

a) Write equations representing the two long-distance plan options.

b) If Charlie uses about 50 minutes of long distance per month, which option is cheaper?

c) How many minutes of long distance would Charlie need to use for both plans to be the same price?

Make sure you know all the details of your cell phone plan, otherwise you may find yourself facing unexpected long-distance or data charges when your bill arrives.

SOLUTION

a) Option 1 is a direct linear relation.
cost = 0.25 × (number of minutes)

Option 2 is a partial linear relation.
cost = 0.10 × (number of minutes) + 5

b) Calculate the cost of each option if 50 minutes are used.

Option 1:

cost = 0.25 × (number of minutes)

cost = 0.25 × 50

cost = $12.50

Option 2:

cost = 0.10 × (number of minutes) + 5

cost = 0.10 × 50 + 5

cost = 5 + 5

cost = $10.00

If Charlie uses 50 minutes of long distance on his cell phone, option 2 is cheaper.

c) Find the number of minutes when the cost of the two plans would be the same. Let n be the number of minutes.

$$\begin{aligned}
\text{cost of option 1} &= \text{cost of option 2} \\
0.25n &= 0.10n + 5 \\
0.25n - 0.10n &= 5 \\
0.15n &= 5 \\
\frac{0.15n}{0.15} &= \frac{5}{0.15} \\
n &= \frac{5}{0.15} \\
n &\approx 33
\end{aligned}$$

Charlie would have to use 33 minutes of long-distance time on his cell phone for the costs of the two plans to be the same.

BUILD YOUR SKILLS

13. Gina is a car salesperson. She earns $750.00/month plus 4% commission on her total sales.

a) Complete the following table of values showing Gina's income based on the following sales totals.

GINA'S EARNINGS BASED ON TOTAL SALES	
Sales	*Earnings*
$10 000.00	
$20 000.00	
$30 000.00	

b) Write an equation that shows the relationship between Gina's income and her sales totals.

c) How much would Gina earn if she sold $100 000.00 worth of cars?

d) If her basic salary was reduced to $600.00/month, but her commission was raised to 5%, would Gina make more or less on $100 000.00 in sales, and by how much?

e) What would Gina's sales have to be for her to earn the same amount under each pay rate?

14. Julien is ordering sandwiches for a lunch in his office. There will be 22 people at the lunch. Julien asks two cafés for the cost of catering the lunch.

Café A will charge $6.50 per sandwich. This includes delivery.

Café B will charge $5.25 per sandwich, plus a delivery fee of $10.00.

a) Write equations representing the lunch options.

b) How much will lunch cost from each of the cafés?

c) How many sandwiches would Julien have to order for the total cost to be the same from both cafés?

PRACTISE YOUR NEW SKILLS

1. Koreen earns $14.75 per hour working as a receptionist.

a) Write an equation that describes the relationship between Koreen's hours of work and her total earnings.

b) What is the slope of the equation? What does it represent?

c) How much will Koreen earn in a 35-hour work week?

2. The table below shows the cost of different sized packages of macadamia nuts. Does the data represent a direct linear relation? Why or why not? If it does, write an equation that shows the relationship between pounds of macadamia nuts and total cost.

MACADAMIA NUT PACKAGE COSTS	
Weight of package (lb)	*Cost*
1	$14.99
2	$29.98
5	$72.55
7	$101.50

3. Canada geese migrate annually. A particular flock of geese was found to have travelled 350 kilometres in 5 hours.

Scientists have tracked Canada geese that have flown up to 1000 km in one day durnig their migration south for the winter.

a) If they flew at a fairly steady rate, what was their approximate speed?

b) Complete the table of values showing the geese's distance from their starting point by elapsed time.

MIGRATING GEESE'S DISTANCE FROM STARTING POINT, BY TIME ELAPSED	
Time elapsed (h)	*Distance flown (km)*
0	
1	
2	
3	
4	
5	

c) Write an equation to represent their distance from their starting point at any flying time.

d) Calculate how many hours it would take them to travel 3000 kilometres.

4. A company is making custom jerseys for a basketball team. The company charges a flat fee of $100.00 to set up the design, plus $75.0 per uniform.

a) Make a table of values to represent the cost of 10, 15, and 20 uniforms.

COST OF BASKETBALL UNIFORM ORDER	
Number of uniforms	*Total cost*
10	
15	
20	

b) Draw a graph of the data.

c) What is the slope of the graph? What does it represent?

d) What is the *y*-intercept? What does it represent?

e) Write an equation to represent the relationship between the total cost and the number of uniforms ordered.

5. Raj's car consumes approximately 6.8 litres of fuel per 100 kilometres of driving.

a) The tank holds 45 litres of fuel. Complete the table below showing the fuel remaining according to distance driven, starting from a full tank.

FUEL REMAINING IN CAR, BY DISTANCE DRIVEN

Distance driven (km)	*Fuel remaining (L)*
0	
100	
200	
300	

b) Make a graph of the data.

c) How much fuel is in the car after 550 km of driving?

d) How far would the car have to be driven to completely empty the fuel tank?

e) Write an equation that represents the relationship between the distance driven and the amount of fuel remaining in the fuel tank.

Scatterplots and Linear Trends 1.3

In the previous section, you have seen that it is often useful to draw a graph to help you analyze and understand data. A **scatterplot** is a particular type of graph used to discover what relationship, if any, exists between two variables.

scatterplot: a graph of ordered pairs, used to evaluate what type of relationship (if any) exists between two variables

correlation: the degree of relatedness between variables

line of best fit: a line drawn on a scatterplot to show the general trend in the data

outlier: a data point that lies significantly outside the general trend of the data

Points that represent the data are drawn on a coordinate plane, then the graph is examined to see if a trend or **correlation** exists. You can examine if the points all seem to cluster around a straight line, or if they are widely spread out all over the graph.

If there appears to be a correlation between the points, you can draw a **line of best fit**. This is a line that shows the general trend in the data. It does not pass through all data points, but instead is drawn so that approximately the same number of points appear above and below it. If the points cluster close to the line of best fit, there is a strong correlation; if they are spread further out, there is a weak correlation.

Using a line of best fit, you can find the following information about the relationship between the variables:

- the shape (linear or non-linear);
- the strength (weak or strong);
- the direction (positive or negative); and
- if any **outliers** exist.

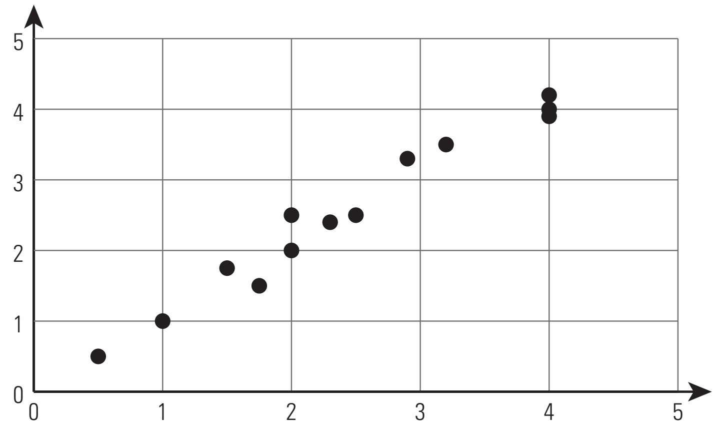
For more information, see page 48 of *MathWorks 12*.

Example 1

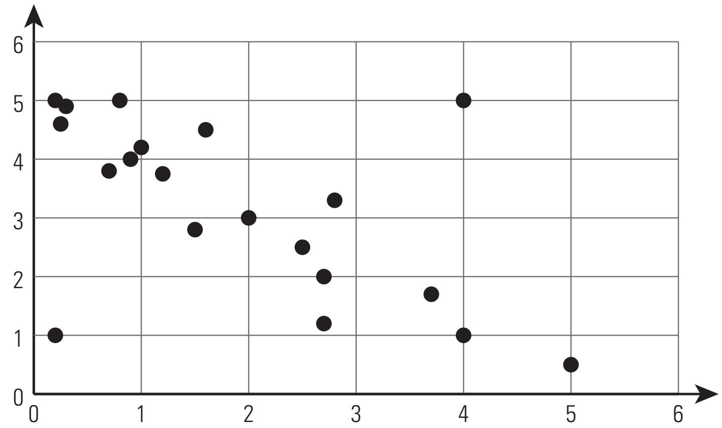
For each of the scatterplots below, describe the relation:

- Is it linear or non-linear? If linear, is it direct or partial?
- What is the direction of the correlation (if any)?
- What is the strength of the correlation?
- Are there any outliers?

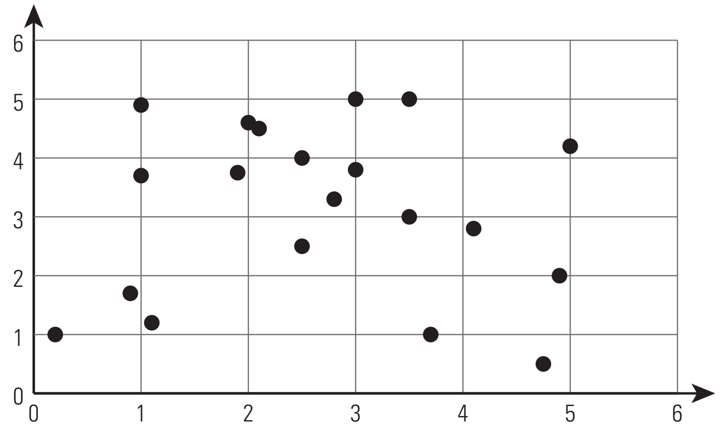
a)


0
1
2
3
4
5
0
1
2
3
4
5

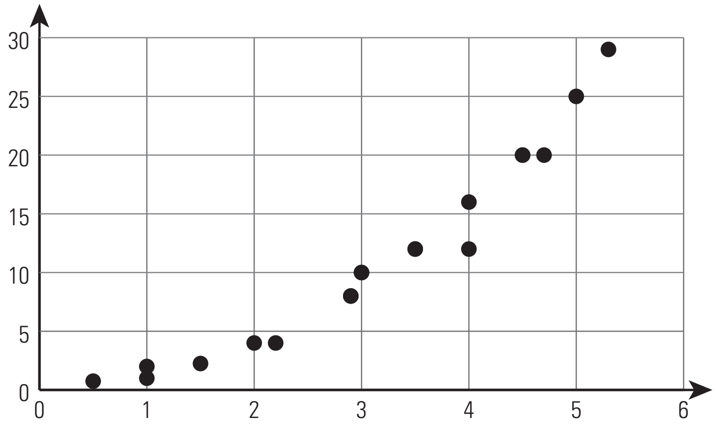
b)


0
1
2
3
4
5
6
0
1
2
3
4
5
6

c)


0
1
2
3
4
5
6
0
1
2
3
4
5
6

d)


0
5
10
15
20
25
30
0
1
2
3
4
5
6

SOLUTION

a)
- The data points seem to form a straight line, so the graph shows a direct linear relation.
- The graph shows a positive slope, so there is a positive correlation.
- The points are quite close together, so the correlation is quite strong.
- There are no outliers.

b)
- The data points seem to form a straight line, but it does not pass through the origin, so the graph shows a partial linear relation.
- The graph shows a negative slope, so there is a negative correlation.
- The points are more spread out than in a). The correlation is moderate.
- There appear to be two outliers (the point at approximately (1.2, 1) and the point at approximately (4, 5)).

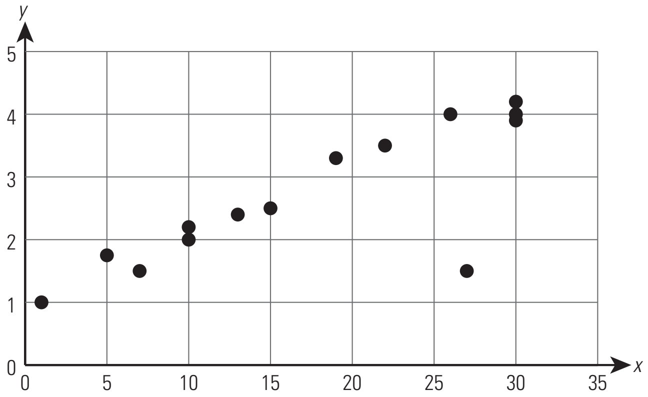
c) There appears to be no relationship between the variables. The data points are all very spread out, and no trend is apparent in the data.

d)
- The data points seem to a non-linear relation.
- There is a positive correlation between the variables.
- The points are quite close together, so the correlation is quite strong.
- There are no outliers.

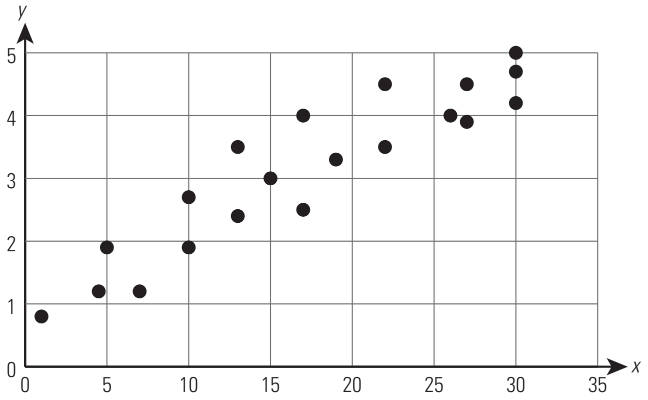
BUILD YOUR SKILLS

1. Examine the two scatterplots below. Identify two similarities and two differences between the graphs.

a)

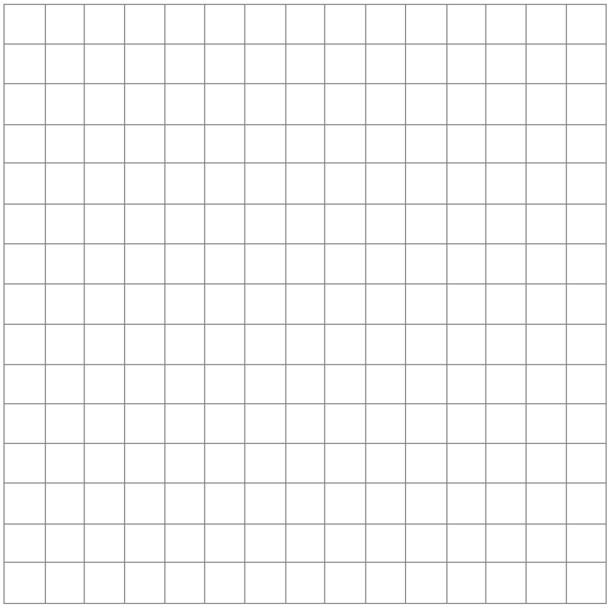


b)



2. a) Draw a scatterplot of the following data.

x	*y*
6	5
5	3
2.5	3.5
0.75	0.5
3	1
5.5	3.25
4	5.5
2	2
4	3.5

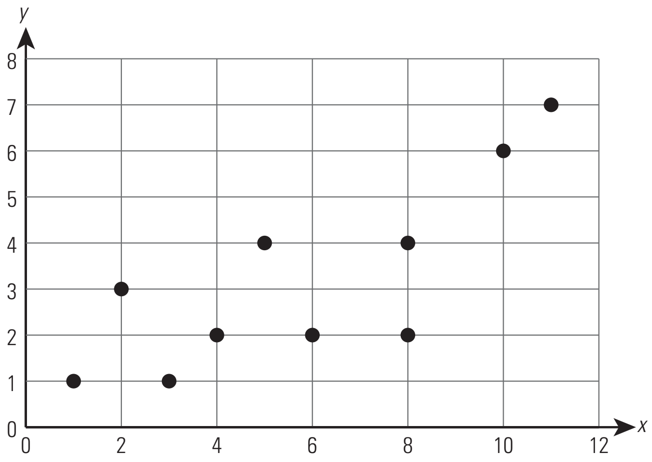


b) Describe the relation between x and y.

- Is it linear or non-linear? If linear, is it direct or partial?
- What is the direction of the correlation (if any)?
- What is the strength of the correlation?
- Are there any outliers?

Example 2

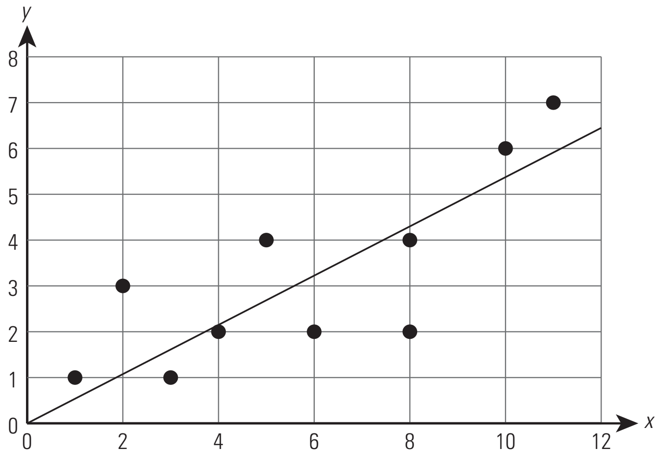
Draw a line of best fit on the scatterplot below, and write an equation for the line.



SOLUTION

The data appears to form a direct linear relation, so a line of best fit can be drawn starting at the origin. The line of best fit does not need to go through any of the data points, but approximately the same number of data points should be above and below the line.

Because drawing the line of best fit is an approximation, your answer may vary slightly from another person's.



Calculate the slope of the line of best fit. Choose a point on the line, such as (2, 1).

$$m = \frac{y}{x}$$

$$m = \frac{1}{2}$$

Write the equation for the direct linear relation.

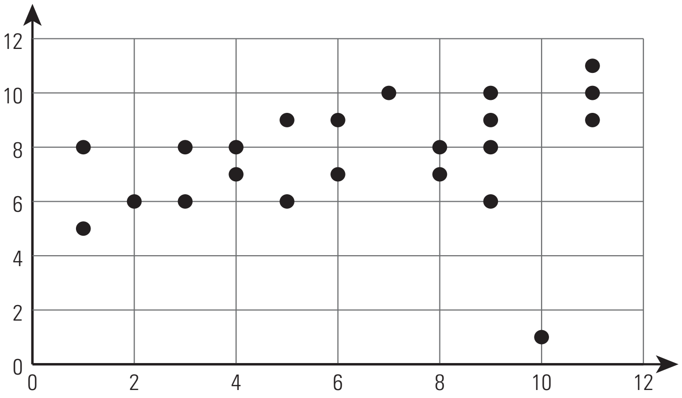
$$y = mx$$

$$y = \frac{1}{2}x$$

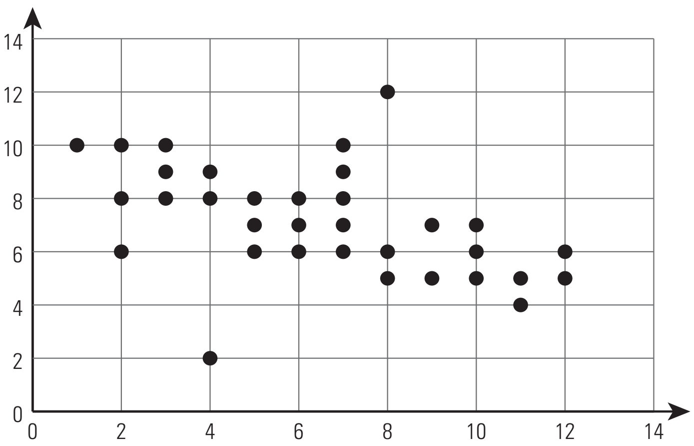
BUILD YOUR SKILLS

3. For each of the following graphs, draw a line of best fit and write an equation to represent it.

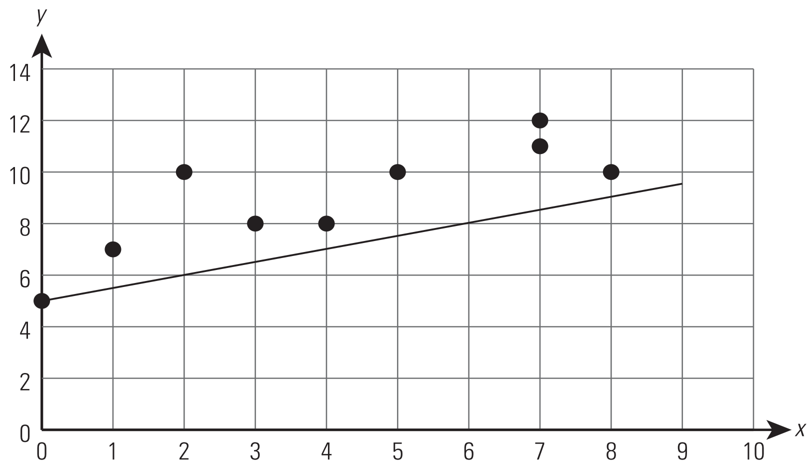
a)



b)



4. a) The scatterplot below shows some collected data points and a line of best fit. With the given data, the line of best fit does not accurately represent the trend. Add 8 data points that would make the drawn line of best fit appropriate.

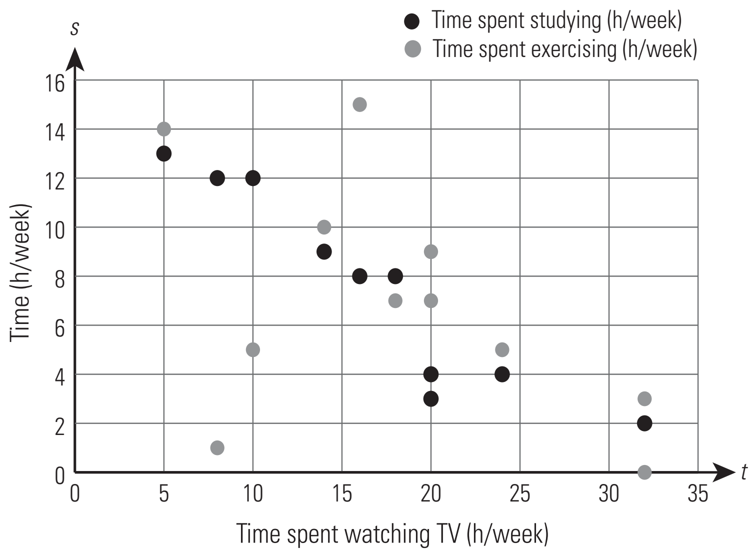


b) Write an equation for the line of best fit.

Example 3

Eadie surveyed students in her class to find out if the time they spent watching TV was related to the time they spent studying for their math exam, or the time they spent exercising in one week. She created the following scatterplot of the results.

Time Spent Watching TV Compared to Studying and Exercising



a) Describe the relationship between time spent watching TV and time spent studying.

b) Describe the relationship between time spent watching TV and time spent exercising.

c) Draw lines of best fit for the two sets of data and write equations for the two lines.

d) Calculate how much time a person would spend studying and exercising if he or she watched TV for 30 hours.

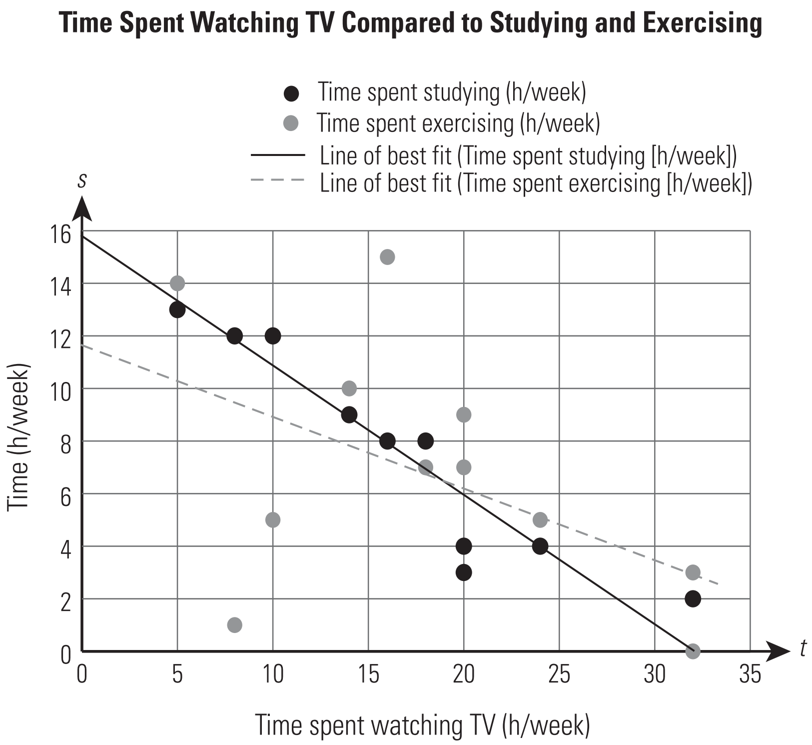
How much time do you spend watching TV in one week? Do you think it affects your study or exercise habits?

SOLUTION

a) The relationship between time spent watching TV and time spent studying appears to be a partial linear relation. The correlation is quite strong, and decreasing.

b) The relationship between time spent watching TV and time spent exercising appears to be a partial linear relation, but the correlation is much weaker than with time spent studying. The correlation is decreasing.

c) Draw lines of best fit on the scatterplot.



For the relationship between time spent watching TV and time spent studying, choose two points on the line of best fit, such as (10, 11) and (20, 6). Calculate the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{6 - 11}{20 - 10}$$

$$m = \frac{-5}{10}$$

$$m = -\frac{1}{2}$$

Calculate the y-intercept using one of the points on the line, such as (20, 6), and the slope.

$$\begin{aligned} y &= mx + b \\ 6 &= -\frac{1}{2}(20) + b \\ 6 &= -10 + b \\ 6 + 10 &= b \\ 16 &= b \end{aligned}$$

The equation of the line is:

$$\begin{aligned} y &= mx + b \\ \text{time spent studying} &= -\frac{1}{2}(\text{time spent watching TV}) + 16 \end{aligned}$$

For the relationship between time spent watching TV and time spent exercising, choose two points on the line of best fit, such as (10, 9) and (20, 6). Calculate the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{6 - 9}{20 - 10}$$

$$m = \frac{-3}{10}$$

$$m = -\frac{3}{10}$$

Calculate the y-intercept using one of the points on the line, such as (10, 9), and the slope.

$$y = mx + b$$
$$9 = -\frac{3}{10}(10) + b$$
$$9 = -3 + b$$
$$9 + 3 = b$$
$$12 = b$$

The equation of the line is:

$$y = mx + b$$
$$\text{time spent exercising} = -\frac{3}{10}(\text{time spent watching TV}) + 12$$

d) Use the equations of the lines of best fit.

$$\text{time spent studying} = -\frac{1}{2}(\text{time spent watching TV}) + 16$$
$$\text{time spent studying} = -\frac{1}{2}(30) + 16$$
$$\text{time spent studying} = -15 + 16$$
$$\text{time spent studying} = 1$$
$$\text{time spent exercising} = -\frac{3}{10}(\text{time spent watching TV}) + 12$$
$$\text{time spent exercising} = -\frac{3}{10}(30) + 12$$
$$\text{time spent exercising} = -9 + 12$$
$$\text{time spent exercising} = 3$$

A person who watches 30 hours of TV per week could be expected to study for 1 h/week and exercise for 3 h/week.

BUILD YOUR SKILLS

5. Alexander wanted to find out if income is related to age. He conducted a survey of two companies' employees' ages and average annual salaries according to that age.

SURVEY RESULTS OF SALARY COMPARED TO AGE		
Age	*Company A* *Annual salary*	*Company B* *Annual salary*
18	$25 000.00	$10 000.00
22	$30 000.00	$22 000.00
23	$27 000.00	$20 000.00
26	$35 000.00	$24 000.00
30	$40 000.00	$32 000.00
34	$48 000.00	$25 000.00
38	$55 000.00	$42 000.00
39	$52 000.00	$45 000.00
42	$60 000.00	$48 000.00
50	$65 000.00	$60 000.00
52	$65 000.00	$50 000.00
58	$70 000.00	$62 000.00
62	$72 000.00	$58 000.00

a) Use the survey data to draw one scatterplot showing the two sets of data.

b) Describe the relationship between the variables. Which company shows a stronger correlation between salary and age?

c) Draw lines of best fit for the two sets of data and write equations for the lines.

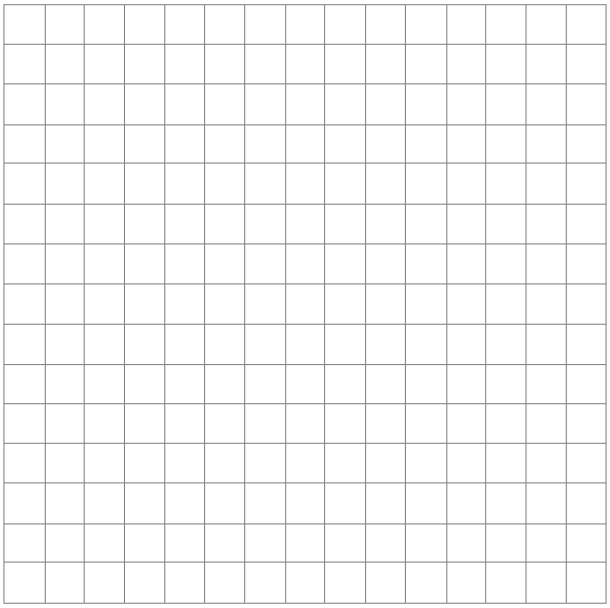
d) Approximately what salary can a 45-year-old employee expect at each of the two companies?

6. The following data show the relationship between men's and women's shoe sizes and their height in inches, for 12 men and 12 women.

MEN'S HEIGHTS COMPARED TO SHOE SIZE	
Shoe size	*Height (inches)*
6.5	60
7	64
11	72
8.5	64
8.5	62
15	76
7	60
6	55
12	72
8	65
11	68
9	78

WOMEN'S HEIGHTS COMPARED TO SHOE SIZE	
Shoe size	*Height (inches)*
5	59
5.5	63
6	60
7	69
7	63
7.5	67
8	65
8	62
8	70
9	69
10	67
12	74

a) Draw one scatterplot showing the two sets of data and their lines of best fit.



b) Does one set of data show a stronger correlation between the variables than the other? Why or why not?

c) According to the Guinness Book of World Records, Robert Wadlow (1918–1940) was the tallest man in recorded history, at just over 107 inches. Assuming the data above is representative of men worldwide, estimate his shoe size.

Robert Wadlow posing with his mother in February 1939.

d) Using the equation for the line of best fit, what would be the shoe size of Chandra Bahadur Dangi (1939–), who, at 21.5 inches tall, is recorded as the world's shortest man? Does this make sense? Why or why not?

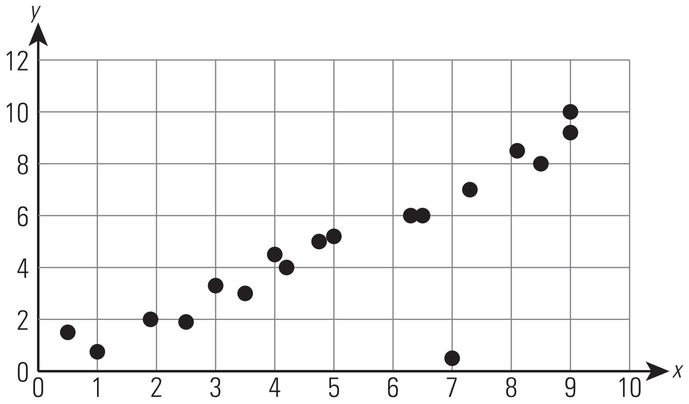
e) The tallest woman in recorded history was Zeng Jinlian (1964–1982), at 97.75 inches. Estimate her shoe size.

PRACTISE YOUR NEW SKILLS

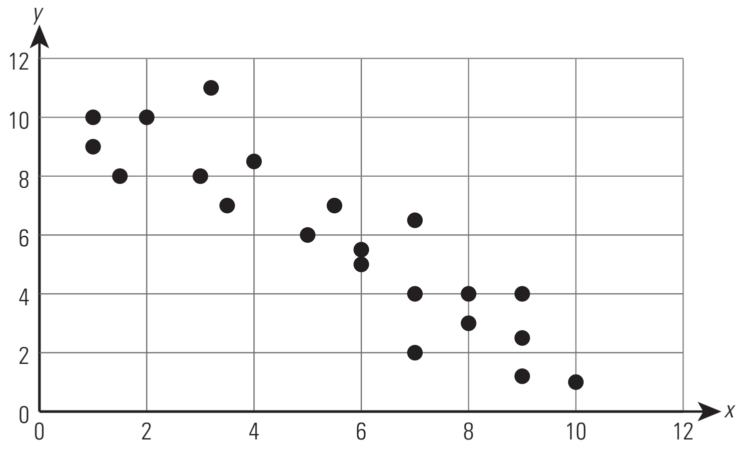
1. For each of the scatterplots below, describe the relation:

   - Is it linear or non-linear? If linear, is it direct or partial?
   - What is the direction of the correlation (if any)?
   - What is the strength of the correlation?
   - Are there any outliers?

a)

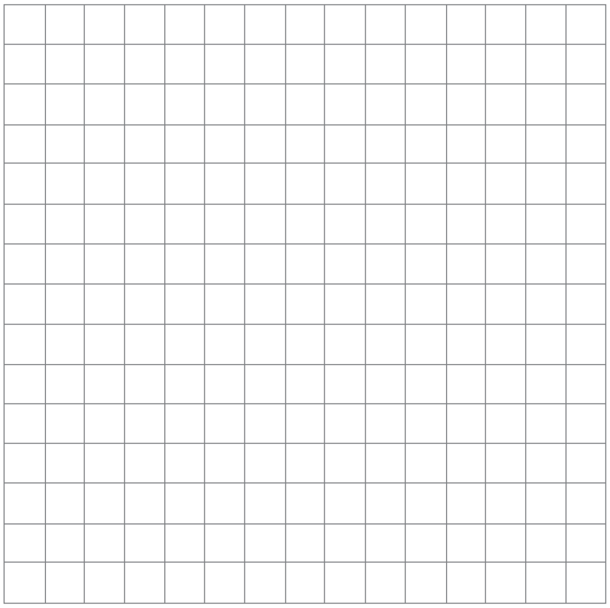


b)



2. a) Draw a scatterplot to show the relationship between the fat content and calories in foods, using the data in the following table. Include a line of best fit.

FAT CONTENT VS. CALORIES IN FOODS	
Fat (g)	*Calories*
10	250
12	300
20	350
30	450
32	500
6	200
6	180
30	500
24	300
16	200

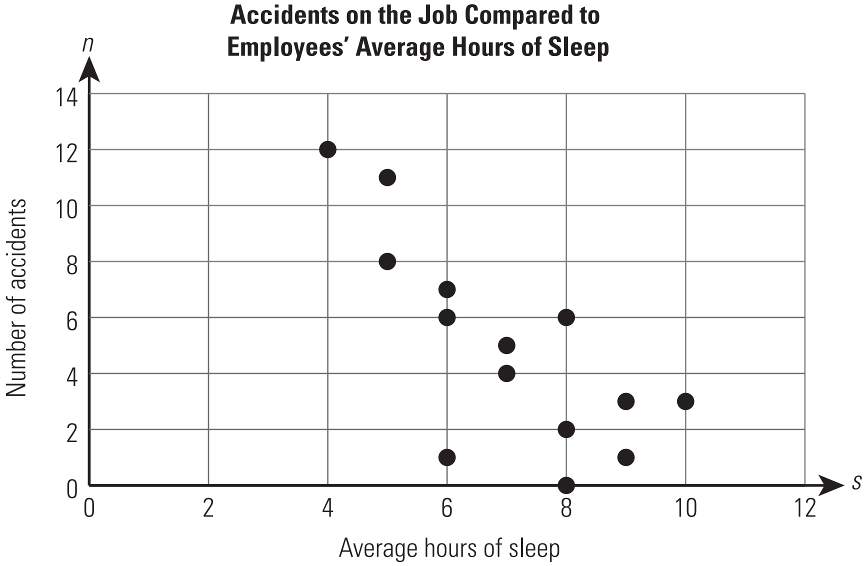


b) Write an equation to represent the relationship between fat content and calories in foods.

c) Using your equation, calculate how many calories a food with 14 g of fat could be expected to have.

3. The following scatterplot shows the relationship between the average number of hours of sleep had by employees compared to the number of accidents on the job over a two-year period.

Do you think you get enough sleep per night, or do you find yourself tired when your alarm clock goes off?



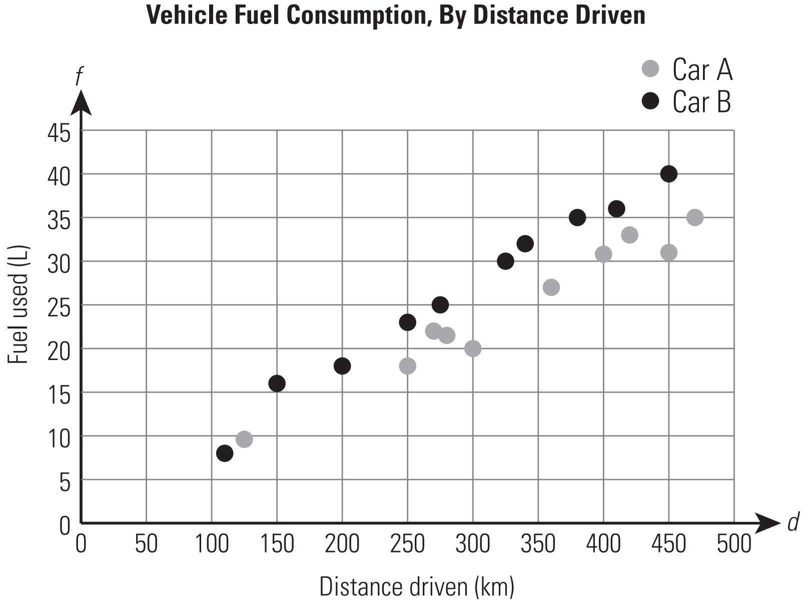
a) Describe the relationship between the variables (if one exists).

b) Draw a line of best fit and write an equation for it.

c) Is it possible for the line of best fit to extend below 0 on the *y*-axis? Why or why not?

4. Gunnar is a vehicle reviewer for a car website. He wants to test the fuel consumption of two popular models that are advertised as fuel efficient.

   He asked 10 drivers of each car model to record how much fuel they used and how far they drove between visits to the gas station. The following scatterplot shows the data Gunnar compiled.



   a) Which vehicle appears to be more fuel efficient? Explain your answer.

   b) Draw lines of best fit for the two sets of data and write equations for the lines.

   c) Using your equations, calculate how far you could drive on 30 L of fuel in each of the cars.

CHAPTER TEST

1. A balloon rises at a rate of 5 m/s.

    a) Create a table of values showing the height of the balloon above the ground for the first 5 seconds after its release and draw a graph of the data.

    b) Write an equation showing the relationship between the height of the balloon and the time elapsed.

    c) How long will it take the balloon to rise to a height of 300 m?

2. A bicycle rental company rents bikes at a rate of $7.50/h.

   a) Write an equation that represents the amount you would have to pay for renting a bike for any given number of hours.

   b) Use the equation to calculate how much it would cost you to rent the bike for 5 hours.

3. Jacqueline is a plumber. She charges a flat rate of $50.00 plus $30.00/h for emergency house calls.

   a) Create a table of values showing Jacqueline's earnings after each hour of work, up to 5 hours. Draw a graph of the data. Describe the relationship between the variables.

Plumbing emergencies can occur at any time of day. Plumbers sometimes have to be available to make repairs in the middle of the night.

b) What is the slope of the graph? What does it represent?

c) What is the y-intercept? What does it represent?

d) Write an equation that describes the relationship between hours of work and earnings.

4. Séan wants to join a gym. There are two gyms in his neighbourhood. Gym A charges a $100.00 fee to join, then charges $19.99 per month of membership. Gym B charges $29.99 per month of membership.

   a) Write equations representing the costs of the two gym memberships.

   b) If Séan plans to keep his membership for 12 months, which gym membership will be cheaper?

   c) How many months would Séan need to be a member for both gyms to cost the same amount?

5. Erika recorded the maximum outside temperature each day for 11 days, and the number of people who went swimming at the outdoor pool where she works.

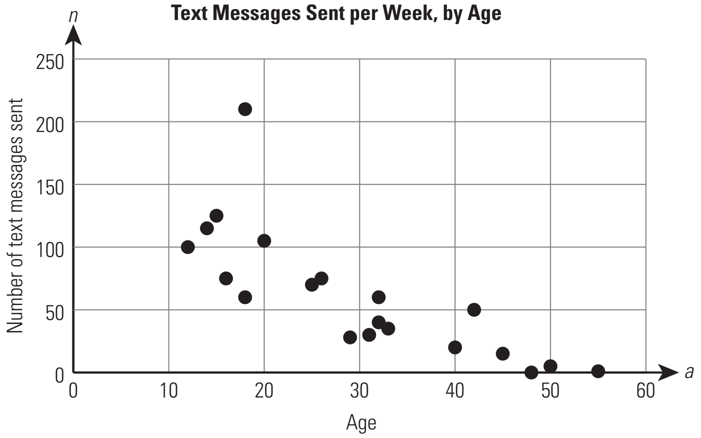
MAXIMUM OUTSIDE TEMPERATURE VS. NUMBER OF SWIMMERS	
Maximum temperature (°C)	*Number of swimmers*
22	155
26	200
21	160
27	280
26	250
26	175
28	300
30	325
30	350
30	375
28	325

a) Draw a scatterplot of the data.

b) What is the relationship between temperature and number of swimmers?

c) If the relationship is linear, draw a line of best fit and write an equation for the line.

d) Using your equation, calculate how many people would be expected to swim if the temperature reached 35°C.



6. The scatterplot on the left shows the results of a survey of 20 people about the number of text messages they sent in a week, recorded by age.

a) Describe the relationship between the variables.

b) Draw a line of best fit and write an equation for the line.

c) Using your equation, how many text messages would you expect a 35-year-old to send per week?

d) How old would you estimate a person to be who sent 100 text messages in a week?

Chapter 2

Limits to Measurement

Working in trades such as construction requires taking many measurements. To build a good quality and durable home, you have to be accurate and precise in your measurements.

2.1 Accuracy and Precision

REVIEW: WORKING WITH THE SYSTÈME INTERNATIONAL

In the Système International (SI), the prefixes tell you the relationships between numbers. For measurements of length, the base unit is the metre (m). The following lengths are equivalent.

UNITS OF LENGTH IN THE SYSTÈME INTERNATIONAL						
millimetre (mm)	*centimetre (cm)*	*decimetre (dm)*	*metre (m)*	*decametre (dam)*	*hectometre (hm)*	*kilometre (km)*
1000	100	10	1	0.1	0.01	0.001

One metre is equal to 1000 mm or 0.001 km.

One kilometre is equal to 1000 m.

Example 1

Canada uses SI units of measurement, while the USA uses imperial units such as feet and miles. You might see a sign like this when crossing the border from the USA to Canada, reminding you of the difference between the systems of measurement.

Use the table of prefixes on the previous page to calculate the following conversions.

a) 65 km = __________ m

b) 1200 m = __________ km

c) 400 dm = __________ m

d) 5600 mm = __________ hm

e) 98 000 000 000 mm = __________ km

SOLUTION

Use the conversion factors given above to do the calculations.

a) $65 \text{ km} \times \dfrac{1000 \text{ m}}{1 \text{ km}} = 65\,000 \text{ m}$

b) $1200 \text{ m} \times \dfrac{1 \text{ km}}{1000 \text{ m}} = 1.2 \text{ km}$

c) $400 \text{ dm} \times \dfrac{1 \text{ m}}{10 \text{ dm}} = 40 \text{ m}$

d) $5600 \text{ mm} \times \dfrac{1 \text{ hm}}{100\,000 \text{ mm}} = 0.056 \text{ hm}$

ALTERNATIVE SOLUTION

You can also do the conversion in multiple steps.

$$5600 \text{ mm} \times \frac{1 \text{ m}}{1000 \text{ mm}} = 5.6 \text{ m}$$

$$5.6 \text{ m} \times \frac{1 \text{ hm}}{100 \text{ m}} = 0.056 \text{ hm}$$

e) $98\,000\,000\,000 \text{ mm} \times \dfrac{1 \text{ km}}{1\,000\,000 \text{ mm}} = 98\,000 \text{ km}$

ALTERNATIVE SOLUTION

Do the conversion in multiple steps.

$$98\,000\,000\,000 \text{ mm} \times \frac{1 \text{ m}}{1000 \text{ mm}} = 98\,000\,000 \text{ m}$$

$$98\,000\,000 \text{ m} \times \frac{1 \text{ km}}{1000 \text{ m}} = 98\,000 \text{ km}$$

BUILD YOUR SKILLS

1. Calculate the following conversions.

   a) 180 cm = __________ m

   b) 76 000 mm = __________ m

   c) 8400 m = __________ km

   d) 96.8 dm = __________ mm

2. Convert the values in the table into each of the other SI units of measure.

UNITS OF LENGTH IN THE SYSTÈME INTERNATIONAL						
millimetre (mm)	*centimetre (cm)*	*decimetre (dm)*	*metre (m)*	*decametre (dam)*	*hectometre (hm)*	*kilometre (km)*
				280		
325.5						
						1.785

REVIEW: WORKING WITH ROUNDING

It is often necessary to round measured values, to show the level of accuracy of your measurements and calculations.

Example 2

Round each measurement to the indicated degree of precision.

a) 87.253 mm to the nearest tenth of a unit

b) 3564 m to the nearest 10

c) 42.3979 cm to the nearest tenth and to the nearest hundredth

SOLUTION

a) Rounding to the nearest tenth of a unit means to 1 decimal place. Look at the number to the right of the tenth position:

   87.2<u>5</u>3 mm

   If it is a digit from 0 to 4, the number will be rounded down. If it is from 5 to 9, the number will be rounded up.

   In this case, 87.2<u>5</u>3 will be rounded up to 87.3 mm. That is, 87.253 mm is closer to 87.3 mm than it is to 87.2 mm.

b) To round to the nearest 10, look at the number to the right of the tens position:

356<u>4</u> m

Since it is a 4, the number is rounded down to 3560 m. This means that 3564 m is closer to 3560 m than it is to 3570 m.

c) First, round to the nearest tenth by looking at the number to the right:

42.3<u>9</u>79 cm

The number to the right is 9, so round up to 42.4 cm.

Next, round to the nearest hundredth by looking at the number to the right of the hundredths position:

42.39<u>7</u>9 cm

The number to the right is 7, so the 9 in the hundredths position must be rounded up to 10—this makes 39 round up to 40, so the number becomes 42.40 cm. You must keep the 0 in the hundredths position to indicate the position to which you have rounded.

BUILD YOUR SKILLS

You have to choose an appropriate measuring device for the item to be measured. A flexible tape measure marked in millimetres or inches would be more appropriate than a metre stick when sewing clothes.

3. Round the following measurements to the nearest tenth of a unit.

a) 567.273 mm
b) 65.681 cm
c) 2.236 m
d) 0.0578 km

4. Round the following measurements to the nearest hundredth of a unit.

a) 92.786 cm
b) 6250.074 m
c) 0.8994 km
d) 640.4199 hm

5. Round to the nearest whole unit.

a) 2987.786 km
b) 12.65 m
c) 5.98 cm
d) 0.087 m

NEW SKILLS: WORKING WITH ACCURACY

In everyday life, people often use the words “precise” and “accurate” to mean the same thing. In mathematics and in trades, however, they have different meanings.

accuracy: how close a measured value is to the true value

Accuracy refers to the exactness of measurement. To be accurate, a measured value must be as close to the real value as possible. If you have measured something accurately, someone should be able to repeat the same measurement and get the same value.

For more details, see page 78 of *MathWorks 12*.

Example 3

Explain if the following measurements would give an accurate measure and if the measuring device is appropriate. State why or why not.

a) Using the length of your forearm to measure the length of a picnic table.

b) Using the length of your stride to mark off the playing field for soccer.

c) Using a metre stick to calculate the distance between two cities.

d) Using a ruler marked in millimetres to measure the width of a book.

When taking measurements, it is important to think about the appropriateness of your unit of measurement. For example, you would not need to know the distance between cities in millimetres. For this measurement, kilometres would be more appropriate.

SOLUTION

a) Using your forearm to measure the length of a picnic table would not be very accurate, because different people have different lengths of forearms. Because the unit (a forearm) cannot be divided into smaller units without guessing, you would not get an accurate measure of the table, so the unit is not appropriate.

b) Using your stride to make a measurement would not be very accurate, because stride length varies from person to person, and even between one person’s own strides. However, strides could be used to make an estimate of a soccer field’s length.

c) Using a metre stick could be accurate, because you are using a standard measuring device. However, a metre stick is not an appropriate unit for measuring this distance.

d) Using a ruler marked in millimetres would give an accurate measurement of the width of a book, because it is a standard measuring device and has an appropriate scale.

BUILD YOUR SKILLS

6. Would it be possible to accurately tell the time on a watch face that has no numbers? Why or why not? Use the following diagram to help explain your answer.

7. Rex stated: "It takes 10 minutes to get from my house to Jordan's house." Explain whether or not this statement is accurate.

8. Is it possible to accurately measure the volume of a pile of dried leaves in the fall? Explain why or why not.

NEW SKILLS: WORKING WITH PRECISION AND UNCERTAINTY

precision: the smallest unit of measurement on the measuring device being used (for example, millimetres on a metre stick)

uncertainty: the margin of error of a measurement; if not stated otherwise, uncertainty is half the precision of the measuring device used

Like accuracy, **precision** is related to exactness, but it is determined by the limitations of the measuring instrument. For example, if your measuring device is marked off in centimetres, that is as precise as you can measure: your answer will be to the nearest centimetre. A measurement made with a ruler marked in millimetres would be more precise than a measurement made with a ruler marked in centimetres.

Because of the limited precision of a measuring device, there is always some **uncertainty** when taking measurements. For example, if you are measuring a box with a ruler marked in centimetres, and it is slightly less than 6 cm, you have to say that it is 6 cm long. You cannot give a more precise measure than this because of the units of the measuring device.

It is often necessary to record how much uncertainty exists in a measurement. Unless stated otherwise, uncertainty is given as half of the precision on the measuring device. For the box example given above, the length would be 6 cm with a precision of 1 cm and an uncertainty of 0.5 cm.

Measurement uncertainty can be written as follows:

measured value ± measurement uncertainty

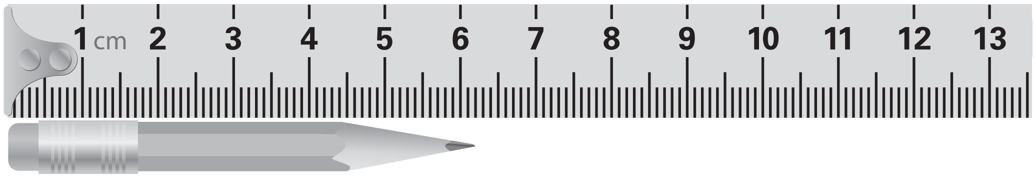
In this form, the length of the box would be 6 cm ± 0.5 cm, that is, between 5.5 cm and 6.5 cm.

For more details, see page 78 of *MathWorks 12*.

Example 4

State the precision and uncertainty of the following measurements.

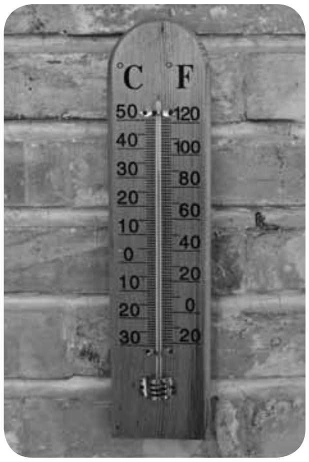
a) A ruler marked in millimetres is used to measure a pencil as shown.



b) An odometer records distance travelled to the nearest kilometre.

c) The outside temperature was recorded as 12°C.

d) Harpreet says it took him about 5 hours to drive from Vancouver to Kamloops, BC.



The smallest unit on the measuring device used determines the precision of a measurement. This thermometer would allow a precision of 1°C or 2°F.

SOLUTION

a) The smallest unit of measurement on the ruler is millimetres. The precision is 1 mm. The uncertainty is half the precision, or ± 0.5 mm.

b) The odometer records distances to the nearest kilometre, so the precision is 1 km. The uncertainty is ± 0.5 km.

c) Because the temperature is not recorded with any decimals (for example, 12.3°C), you can assume that the thermometer recorded the temperature to the nearest degree. The precision is 1°C, and the uncertainty is ± 0.5°C.

d) You cannot be certain of the precision, because Harpreet said "*about* 5 hours." There are many possibilities, including:

- If he rounded to the nearest hour, the precision would be 1 h and the uncertainty would be ± 0.5 h (or 30 min).
- If he rounded to the nearest minute, the precision would be 1 min and the uncertainty would be ± 0.5 min (or 30 sec).
- If Harpreet rounded the time to the nearest quarter hour (15 minutes), the precision would be 15 min and the uncertainty would be ± 7.5 min.

BUILD YOUR SKILLS

9. Runa used a metre stick marked off in centimetres to measure the length of the classroom. What are the precision and the uncertainty?

10. The odometer on Sam's car measures distances to the nearest tenth of a kilometre. What are the precision and the uncertainty of the odometer?

11. Ta-Ming is a veterinary nurse. He measured a dog's temperature to be 38.8°C. What are the precision and the uncertainty?

Example 5

Geneviève is mixing juice in a jug marked in quarter-litres. She measures the amount of juice in the jug to be 2.5 L.

a) Write the measurement of juice in the jug in the form:
measured value ± measurement uncertainty.

b) What is the range of volume (in litres) that could be in the jug?

SOLUTION

a) The jug is marked in quarter-litres (0.25 L). This is the precision. The uncertainty is half the precision.

0.25 L ÷ 2 = 0.125 L

The amount of juice in the jug is 2.5 L ± 0.125 L.

b) The maximum amount of juice that could possibly be in the jug is the measured value plus the uncertainty.

2.5 L + 0.125 L = 2.625 L

The minimum amount is the measured value minus the uncertainty.

2.5 L – 0.125 L = 2.375 L

There could be between 2.375 L and 2.625 L of juice in the jug.

BUILD YOUR SKILLS

12. Frank measured his height as 1.75 m using a tape measure marked in centimetres.

a) Write his height as: measured value ± measurement uncertainty.

b) What is the range of possible values of his actual height?

In Canada, ovens usually show temperature in degrees Fahrenheit, even though we use SI units for most measurements. In Europe, ovens usually show temperature in degrees Celsius.

13. Marcia's oven indicates temperatures to the nearest 10°C. She sets it at 160°C.

    a) What are the precision and uncertainty of the oven thermometer?

    b) What is the maximum possible temperature in the oven? What is the minimum possible temperature?

14. Canadian swimmer Annamay Pierse recorded a time of 2:20.12 (recorded as min:sec) in the 200-metre breaststroke.

    a) What is the precision of measurement?

    b) What is the range of possible values for her actual time?

NEW SKILLS: ADDING AND SUBTRACTING VALUES WITH UNCERTAINTIES

When adding or subtracting measurements that each have an uncertainty, the combined value will also have an uncertainty.

The uncertainty of the final amount is the sum of the measured uncertainties.

For more details, see page 84 of *MathWorks 12*.

Example 6

Marcie is a carpenter. She is installing sheets of plywood as subflooring on a balcony. She has measured two of the sheets to be 2.25 m and 1.30 m long, using a tape measure marked in centimetres.

a) Write the lengths of each sheet of plywood in the form: measured value ± measurement uncertainty.

b) What is the total length, including uncertainty, of the two sheets if they are placed end to end?

c) Marcie needs to shorten the 2.25-m sheet of plywood. She cuts a 30-cm section from the end. What is the new length of this sheet, including uncertainty?

SOLUTION

a) Marcie measured the plywood using a tape measure marked in centimetres, so the precision of each measurement is 1 cm. The uncertainty is half the precision, or ± 0.5 cm. The sheets of plywood have the following dimensions:

2.25 m ± 0.5 cm, or 225 cm ± 0.5 cm

1.30 m ± 0.5 cm, or 130 cm ± 0.5 cm

b) Calculate the total length of the measured values.

total length = 225 + 130

total length = 355 cm

Calculate the sum of the uncertainties.

total uncertainty = 0.5 + 0.5

total uncertainty = 1 cm

The total length of the two sheets of plywood is 355 cm ± 1 cm. This means that the combined length could be anywhere between 354 cm and 356 cm.

c) The sheet of plywood started off at a length of 225 cm ± 0.5 cm. The section that was cut off has an uncertainty of ± 0.5 cm, because it was measured using the same tape measure. The cut section's length can be written as 30 cm ± 0.5 cm.

Calculate the difference between the measured values.

225 – 30 = 195 cm

Add the uncertainties.

0.5 + 0.5 = 1 cm

The new length of the shortened sheet of plywood is 195 cm ± 1 cm.

BUILD YOUR SKILLS

15. Craig and Stephy's wedding rings are being made by a jeweller. Craig's ring weighs 9.22 g and Stephy's weighs 5.73 g.

    a) What are the precision and uncertainty for the weight of each ring?

    b) What are the maximum and minimum possible combined weights of the two rings?

A backyard garden can be a great source of nutritious and delicious vegetables.

16. Colm is preparing liquid fertilizer to apply to his garden. He has filled a bucket marked in half-litres with 4 L of the fertilizer mixture.

    a) Write the volume of liquid in the bucket in the form: measured value ± measurement uncertainty.

    b) Colm uses a small jug marked in 125-mL increments to remove 375 mL of the mixture from the bucket. Write the volume of liquid in the small jug in the form: measured value ± measurement uncertainty.

    c) What is the volume of liquid left in the bucket? Write your answer in the form: measured value ± measurement uncertainty.

17. Danita wants to install new cupboards in her kitchen. The space available in her kitchen is 130 cm wide, measured to the nearest centimetre. She plans to buy pre-built cupboards. The sizes available are 41 cm, 65 cm, and 24 cm wide. If Danita were to buy one of each size of the cupboards, can she safely assume that they will fit in the available space? Why or why not?

If you do not take measurement uncertainty into account in a task such as cupboard installation, you may find yourself with cupboards that are too big for the available space.

PRACTISE YOUR NEW SKILLS

1. Convert the values in the table into each of the other SI units of measure.

UNITS OF LENGTH IN THE SYSTÈME INTERNATIONAL						
millimetre (mm)	*centimetre (cm)*	*decimetre (dm)*	*metre (m)*	*decametre (dam)*	*hectometre (hm)*	*kilometre (km)*
			123.1			
	315.9					
						29.15

2. Round each measurement to the indicated degree of precision.

   a) 1.0925 km to the nearest hundredth

   b) 29.51 cm to the nearest whole unit

   c) 6021.139 mm to the nearest tenth

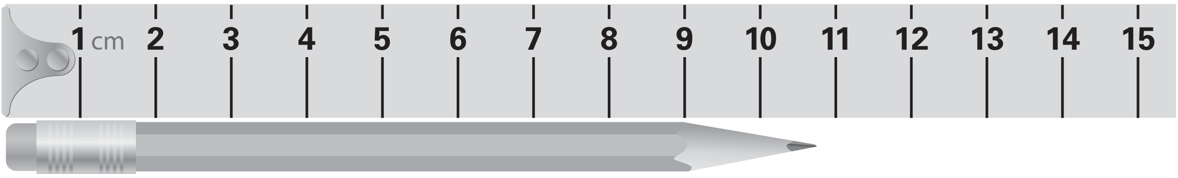
3. Would you expect the following methods to give accurate measurements? Why or why not?

   a) Measuring the amount of water in a tub using a tablespoon.

b) Measuring the thickness of a piece of sheet metal using a metre stick.

c) Determining the heat of a liquid using a thermometer.

4. Examine the following measurement:



a) What is the precision of the measuring device?

b) What is the length of the measured item, including its uncertainty?

5. Alyzeh is a nurse, and she is filling a syringe with medicine for a patient. The dose required is 24 mL. The syringe is marked in 2 mL units.

a) What are the precision and uncertainty of Alyzeh's measurement?

b) What are the maximum and minimum amounts of medicine that could be in the syringe?

6. Jacob went fishing and caught three salmon. He weighed the fish on a scale, and found them to weigh 9.3 kg, 4.7 kg, and 8.4 kg.

   a) What assumption can you make about the precision of the scale?

   b) What is the uncertainty for each of the fish weights?

   c) What is the maximum combined weight of the three fish? What is the minimum combined weight?

People come from around the world to experience salmon fishing on Canada's west coast. This area is home to sockeye, pink, chum, chinook, and coho salmon.

7. Roxana is completing an art project. She has a spool of ribbon that is 15.5 m long. She cuts off two pieces of ribbon that are 3.8 m and 4.2 m long. How much ribbon is left on the spool, assuming the precision of the measuring device is 1 decimetre? Remember to include the measurement uncertainty.

2.2 Tolerances

REVIEW: ADDING AND SUBTRACTING FRACTIONS

In this section, you will need to add and subtract fractions and mixed numerals.

Example 1

Complete the following operations.

a) $\frac{1}{16} + \frac{3}{4}$

b) $\frac{15}{16} - \frac{3}{4}$

c) $3\frac{1}{2} + \frac{1}{8}$

d) $2\frac{5}{8} - \frac{1}{2}$

SOLUTION

When adding and subtracting fractions and mixed numerals, remember to work with common denominators.

a)

$$\frac{1}{16} + \frac{3}{4} = \frac{1}{16} + \left(\frac{3 \times 4}{4 \times 4}\right)$$

Convert $\frac{3}{4}$ to a fraction with a denominator of 16.

$$\frac{1}{16} + \left(\frac{3 \times 4}{4 \times 4}\right) = \frac{1}{16} + \frac{12}{16}$$

$$\frac{1}{16} + \frac{12}{16} = \frac{13}{16}$$

Add the numerators.

b)

$$\frac{15}{16} - \frac{3}{4} = \frac{15}{16} - \left(\frac{3 \times 4}{4 \times 4}\right)$$

Convert $\frac{3}{4}$ to a fraction with a denominator of 16.

$$\frac{15}{16} - \left(\frac{3 \times 4}{4 \times 4}\right) = \frac{15}{16} - \frac{12}{16}$$

$$\frac{15}{16} - \frac{12}{16} = \frac{3}{16}$$

Subtract the numerators.

c) $3\frac{1}{2} + \frac{1}{8} = \left(\frac{6}{2} + \frac{1}{2}\right) + \frac{1}{8}$ Convert $3\frac{1}{2}$ to an improper fraction.

$\left(\frac{6}{2} + \frac{1}{2}\right) + \frac{1}{8} = \frac{7}{2} + \frac{1}{8}$

$\frac{7}{2} + \frac{1}{8} = \left(\frac{7 \times 4}{2 \times 4}\right) + \frac{1}{8}$ Convert $\frac{7}{2}$ to a fraction with a denominator of 8.

$\left(\frac{7 \times 4}{2 \times 4}\right) + \frac{1}{8} = \frac{28}{8} + \frac{1}{8}$

$\frac{28}{8} + \frac{1}{8} = \frac{29}{8}$

$\frac{29}{8} = 3$, remainder 5 Divide.

$\frac{29}{8} = 3\frac{5}{8}$

d) $2\frac{5}{8} - \frac{1}{2} = \left(\frac{16}{8} + \frac{5}{8}\right) - \frac{1}{2}$ Convert $2\frac{5}{8}$ to an improper fraction.

$\left(\frac{16}{8} + \frac{5}{8}\right) - \frac{1}{2} = \frac{21}{8} - \frac{1}{2}$

$\frac{21}{8} - \frac{1}{2} = \frac{21}{8} - \left(\frac{1 \times 4}{2 \times 4}\right)$ Convert $\frac{1}{2}$ to a fraction with a denominator of 8.

$\frac{21}{8} - \left(\frac{1 \times 4}{2 \times 4}\right) = \frac{21}{8} - \frac{4}{8}$

$\frac{21}{8} - \frac{4}{8} = \frac{17}{8}$

$\frac{17}{8} = 2$, remainder 1 Divide.

$\frac{17}{8} = 2\frac{1}{8}$

BUILD YOUR SKILLS

1. Complete the following operations.

a) $\frac{1}{2} + \frac{1}{8}$

b) $2\frac{1}{4} + \frac{1}{16}$

c) $3\frac{11}{16} - \frac{5}{8}$

d) $\frac{7}{8} - \frac{3}{16}$

e) $5\frac{1}{2} + 3\frac{7}{8}$

f) $3\frac{3}{4} - \frac{1}{16}$

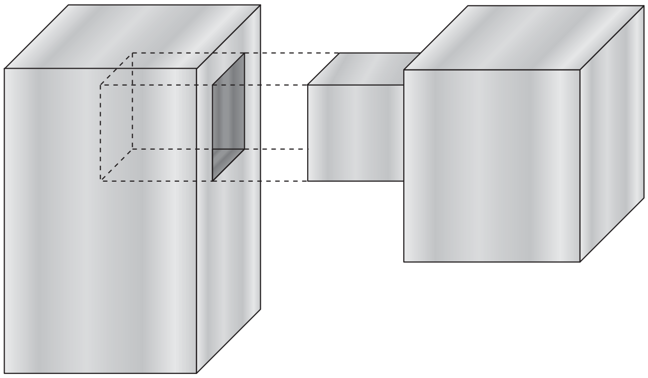
NEW SKILLS: WORKING WITH TOLERANCE

tolerance: the measurement variation allowed so that an item is acceptable for its intended purpose

nominal value: the target value for a measurement

When building or assembling any object, from a simple bookcase to a complex computer, you must take measurements. Your measurements should always be as accurate as possible, and they should be precise enough for the needs of the project.

When manufacturing a product, a range of sizes in often acceptable. For example, if you are making a mortise and tenon joint as shown in the diagram, the tenon needs to be small enough to fit in the mortise, but large enough that it will not fall out. The range in acceptable values is called a **tolerance**.



The tolerance is determined by subtracting the minimum acceptable value from the maximum acceptable value. The target value, usually halfway between the minimum and maximum values, is called the **nominal value**.

A manufacturing tolerance can be written in four ways:

maximum value
minimum value

nominal value $\pm \frac{1}{2}$(tolerance)

minimum value$^{+\text{tolerance}}_{-0}$

maximum value$^{+0}_{-\text{tolerance}}$

For more details, see page 92 of *MathWorks 12*.

Example 2

Mackenzie is making caramel candies. The recipe says to use a candy thermometer to monitor the temperature of the sugar mixture. If it is not heated enough, the candies will not hold their shape, and if it is heated too much, the candies will be too firm. The sugar mixture must be heated to between 245°F and 250°F.

a) What is the temperature tolerance of the sugar mixture?

b) Write the acceptable temperature range in each of the four ways shown above.

> Note that, in some forms, the way a tolerance is expressed looks like a measurement uncertainty. Tolerance and uncertainty are not the same: an uncertainty shows the margin of error in a measured value, while a tolerance shows the range of acceptable values for a measurement.

SOLUTION

a) The tolerance is the difference between the maximum and minimum values.

250°F – 245°F = 5°F

The tolerance is 5°F.

b) The acceptable temperature range in the form $\frac{\text{maximum value}}{\text{minimum value}}$ is $\frac{250°\text{F}}{245°\text{F}}$.

To write it in the form $\text{nominal value} \pm \frac{1}{2}(\text{tolerance})$, you need to calculate half the tolerance. This is called the ± range.

5°F ÷ 2 = 2.5°F

You also need to calculate the nominal value. To do this, add the maximum and minimum values and divide by 2.

$$\text{nominal value} = \frac{\text{maximum} + \text{minimum}}{2}$$

$$\text{nominal value} = \frac{250°\text{F} + 245°\text{F}}{2}$$

$$\text{nominal value} = \frac{495°\text{F}}{2}$$

$$\text{nominal value} = 247.5°\text{F}$$

The acceptable temperature range is 247.5°F ± 2.5°F.

In the form $\text{minimum value}^{+\text{tolerance}}_{-0}$, the temperature range is $245°\text{F}^{+5°\text{F}}_{-0}$.

In the form $\text{maximum value}^{+0}_{-\text{tolerance}}$, the temperature range is $250°\text{F}^{+0}_{-5°\text{F}}$.

BUILD YOUR SKILLS

2. Write the following maximum and minimum values in the form nominal value $\pm \frac{1}{2}$(tolerance).

 a) maximum = 25.75 mm; minimum = 25.25 mm

 b) maximum = 16°C; minimum = 22°C

 c) maximum = 20″; minimum = 19″

A custom-made suit requires taking many measurements. A tailor will measure the person's shoulder width, back length, wrist size, and several other measurements to create a perfect fit.

3. Adam is a tailor. He is sewing a pair of pants for a client. The waist measurement must be between $33\frac{3}{4}''$ and $34\frac{1}{4}''$.

 a) What are the nominal value and the ± range of the waist measurement?

 b) What is the tolerance?

 c) Write the dimension in the form $\text{minimum value}^{+\text{tolerance}}_{-0}$ and $\text{maximum value}^{+0}_{-\text{tolerance}}$.

4. A part being manufactured in a machine shop has a maximum allowable dimension of 16.28 mm and a minimum of 16.16 mm.

   a) What is the manufacturing tolerance?

   b) Write the dimension in the form $\text{nominal value} \pm \frac{1}{2}(\text{tolerance})$.

   c) Write the dimension in the form $^{\text{maximum value}}_{\text{minimum value}}$.

Example 3

Serena is making a new front door for her house. In order to fit properly in the frame, the door width must have a nominal value of 24″ and a tolerance of $\frac{1}{8}''$.

What are the maximum and minimum allowable values of the door width?

What type of measuring device do you think Serena should use to take the required measurements?

SOLUTION

The tolerance indicates that the maximum and minimum values differ by $\frac{1}{8}''$, and that the nominal value is halfway between the maximum and minimum values. So, calculate half the tolerance.

$$\frac{1}{8}'' \times \frac{1}{2} = \frac{1}{16}''$$

The maximum value is the nominal value plus half the tolerance.

$$24'' + \frac{1}{16}'' = 24\frac{1}{16}''$$

The minimum value is the nominal value minus half the tolerance.

$$24'' - \frac{1}{16}'' = 23\frac{15}{16}''$$

BUILD YOUR SKILLS

5. The nominal value of a measurement is 16.056 m with a tolerance of 0.0007 m. Which of the following measurements meet the standard?

ACCEPTABLE MEASURED VALUES	
Measured dimension (m)	*Acceptable? (Yes/No)*
16.056 72	
16.055 757	
16.056 05	
16.055 85	
16.056 25	
16.055 655	
16.055 35	

6. Write the maximum and minimum values represented by each of the following dimensions.

a) 350°F ± 5°F

b) $18'' \pm \frac{1}{8}''$

c) $25 \text{ mm} ^{+0}_{-0.5 \text{ mm}}$

d) $1.175 \text{ cm} ^{+0.050 \text{ cm}}_{-0}$

e) $3\frac{1}{2}'' \; ^{+0}_{-\frac{1}{8}''}$

f) $24\frac{1}{4}'' \; ^{+\frac{1}{8}''}_{-0}$

7. A machinery part has a nominal value of 8.9 cm and a tolerance of 0.005 cm.

   a) Calculate the maximum and minimum acceptable values.

   b) Write the dimension in the form $\text{minimum value}^{+\text{tolerance}}_{-0}$.

Example 4

Christian is building a brick wall as a fence in his clients' yard. His clients have asked that the wall be 2.40 m tall with a tolerance of 10 cm, so as not to obstruct the view from their patio and so that their dog will not be able to jump over the fence.

a) What is the range of acceptable heights of the wall?

b) If each brick has a height of 7.5 cm ± 0.25 cm, and the mortar must be between 0.75 cm and 1 cm thick between each row of bricks, how many rows of bricks and mortar will it take to build the wall?

Bricks can be laid in a variety of patterns, called bonds. In single-thickness walls, the bond serves only a decorative purpose. In double-thickness walls, the bond also serves a structural purpose, strengthening the wall by connecting the layers.

SOLUTION

a) The wall height specified by the clients is the nominal height, 2.40 m. Convert this to centimetres.

2.40 m = 240 cm

Calculate half the tolerance.

10 cm ÷ 2 = 5 cm

The maximum height is the nominal value plus half the tolerance.

240 + 5 = 245 cm

The minimum height is the nominal value minus half the tolerance.

240 – 5 = 235 cm

The wall height must be between 235 cm and 245 cm.

b) Each brick has a height of 7.5 cm ± 0.25 cm. Calculate the maximum and minimum heights of the brick.

Maximum: 7.5 cm + 0.25 cm = 7.75 cm
Minimum: 7.5 cm – 0.25 cm = 7.25 cm

The maximum thickness of the mortar is 1 cm, and the minimum is 0.75 cm.

Calculate the combined maximum and minimum heights of one brick and one layer of mortar.

Maximum: 7.75 cm + 1 cm = 8.75 cm
Minimum: 7.25 cm + 0.75 cm = 8.00 cm

To find the maximum number of layers of bricks Christian could use, divide the maximum height of the wall (245 cm) by the minimum height of a brick and mortar (8.00 cm).

245 cm ÷ 8.00 cm = 30.625

The answer must be rounded down, because 31 layers would make the wall too high (31 × 8.00 cm = 248 cm).

The maximum within the allowable height range is 30 layers of bricks.

To find the minimum number of layers of bricks that could be used, divide the minimum wall height (235 cm) by the maximum brick and mortar height (8.75 cm).

235 cm ÷ 8.75 cm ≈ 26.9

Christian needs a minimum of 26.9 layers, but you must round to the nearest whole number. The minimum within the allowable height range is 27 layers of bricks.

Therefore, for the height to be within the allowable range, the wall will have between 27 and 30 layers of bricks.

BUILD YOUR SKILLS

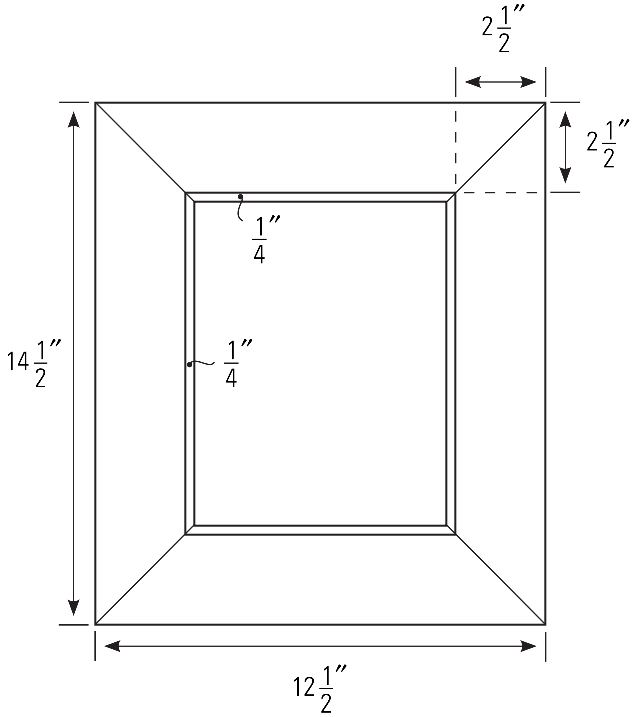
8. An electrical resistor has resistance with a nominal value of 100 ohms and a tolerance of 3%.

   a) What is the range of acceptable values, in ohms, for the resistor?

   b) Write the resistance in the form nominal value $\pm \frac{1}{2}$(tolerance).

9. Darius is cutting glass that will be inserted into the picture frame shown below. The picture frame is $12\frac{1}{2}''$ wide and $14\frac{1}{2}''$ long. It has a $2\frac{1}{2}''$ border that will be visible around the picture inserted into the frame. The glass must sit on a lip that is $\frac{1}{4}''$ wide.

   a) What are the piece of glass's maximum and minimum dimensions, and their tolerances?



b) Would it be better to make the glass larger or smaller? How could you write the tolerance of the glass to show this?

10. Marieta is building a chair. Each chair leg is attached to the chair seat by a peg that fits into a hole in the seat. The nominal diameter of the peg is 1.2 cm with a tolerance of 0.04 cm.

a) Write the diameter of the peg in the form $^{\text{maximum value}}_{\text{minimum value}}$.

b) Explain why there is a tolerance for the diameter of the peg.

PRACTISE YOUR NEW SKILLS

1. Complete the following table.

TOLERANCES				
Nominal value	*Minimum value*	*Maximum value*	*Tolerance*	*± range*
115 mm			6 mm	
	12.5 cm	17.5 cm		
	85°F		10°F	
$3\frac{3}{4}''$				$\frac{1}{8}''$

2. Munira is making custom kitchen cabinets. The front face of a drawer must have a width between $17\frac{15}{16}''$ and $18\frac{1}{16}''$ and a height between $6\frac{15}{16}''$ and $7\frac{1}{16}''$.

   a) What are the nominal dimensions of the drawer face?

   b) What is the tolerance in the measurements?

   c) Write the width and height in the form $\text{nominal value} \pm \frac{1}{2}(\text{tolerance})$.

When installing custom kitchen cabinetry, a cabinetmaker needs to take into account how the client plans to use the space so that she can design a layout that makes the best possible use of the available space.

3. A chemical mixture in a manufacturing process needs to reach a nominal temperature of 10.0°C, with a tolerance of 0.5°C.

   a) What are the minimum and maximum acceptable temperatures? Write them in the form $^{\text{maximum value}}_{\text{minimum value}}$.

   b) Write the temperature in the form $\text{nominal value} \pm \frac{1}{2}(\text{tolerance})$.

4. A medication specifies that it contains 0.0406 mg ± 0.0001 mg of a specified drug. Calculate the tolerance and the maximum and minimum quantities of the drug present in the medication.

5. Junichi is tiling a roof. The building code specifies that roof tile may have an overhang of a maximum of 64 mm and a minimum of 35 mm.

   a) Calculate the nominal overhang for a roof tile.

   b) Express the tolerance in four different ways.

CHAPTER TEST

1. Explain the difference between the following terms. Give an example to support your answer.

   a) accuracy and precision

   b) uncertainty and tolerance

2. Can a non-traditional unit be used to accurately measure the length of an object? Explain why or why not, using an example.

3. Which SI unit of length would be most appropriate for measuring each of the following items?

   a) the distance from Winnipeg, MB, to Regina, SK

   b) the thickness of a coin

   c) the length of a soccer field

   d) the length of a running shoe

4. What are the precision and uncertainty of the following measurements?

   a) 2.5 km

   b) 184 cm

   c) 405.33 kg

   d) 78.1°C

5. Logan is measuring the length of a part being manufactured in his machine shop. Its true measure is 26.025 956 mm. What measurement would Logan record, including the measurement uncertainty, using each of the following measuring devices?

   a) a ruler marked in centimetres

   b) a tape measure marked in millimetres

   c) a ruler marked in half-millimetres

   d) a micrometer, with the smallest gradation of 0.002 mm

6. Mali is following a recipe for a Thai red curry. The recipe calls for $3\frac{1}{2}$ cups of coconut milk. If Mali uses a measuring cup marked in quarter cups to measure the coconut milk, what are the maximum and minimum measures of coconut milk that she will add to the recipe?

7. Aksel is a shipping and receiving clerk at a warehouse. He is preparing packages for shipment to a customer. In one box, he packs items weighing 11.3 kg, 965 g, and 5.6 kg. What are the uncertainties of each of the items' weights? What is the combined weight and uncertainty of the box? (Ignore the weight of the box itself.)

8. Mary is cutting carpet to be installed in a hallway. The carpet is 5.65 m long. She cuts off a piece that is 29 cm long. What is the length, including uncertainty, of the remaining length of carpet?

9. A processing facility packages flour for sale in grocery stores. A 10-kg sack of flour must be within a tolerance of 0.5 kg to be acceptable for sale. What is the acceptable range of weights of a bag of flour?

10. The diameter of a drill hole for a dowel joint is specified as 1.524 cm ± 0.012 cm.

    a) What are the maximum and minimum accepted diameters of the drill hole?

    b) Which of the following drill holes meet the standard?

ACCEPTABLE MEASURED VALUES	
Drill hole diameter (cm)	*Acceptable? (Yes/No)*
1.5241	
1.5119	
1.532	
1.540	
1.5355	
1.537	

11. Ali is a glass lens cutter. He uses a micrometre to measure the glass. In order to fit a particular glass frame, a lens needs to have a thickness of 0.8 mm ± 0.005 mm.

    a) What is the tolerance?

    b) What are the maximum and minimum acceptable lens thicknesses?

The quality of a weld can be tested by checking for cracks, distortions, and bubbles, or by destructive testing to determine the weld's performance under certain conditions.

12. Kelsey is a welder. She needs to preheat a section of steel before welding, to reduce the risk of cracking as the steel cools. She needs to heat the steel to between 225°F and 275°F.

    a) What is the temperature tolerance?

    b) What is the nominal temperature?

    c) Write the acceptable temperature range in four different ways.

Chapter 3

Statistics

Statistical calculations such as mean, median, and mode are important in many businesses to track trends, analyze patterns, and plan for the future.

Mean, Median, and Mode 3.1

REVIEW: ORGANIZING DATA

In this section, you will often have to organize data in ascending (going up) or descending (going down) order. Using a stem and leaf plot is a quick way to organize the data. In a stem and leaf plot, you organize the numbers by tens and arrange them in a table.

Example 1

Write the following numbers in ascending order in a stem and leaf plot.

16, 45, 36, 18, 23, 43, 48, 45, 33, 26, 32, 28, 25, 19, 34, 24, 35, 40, 12, 20, 105, 3

SOLUTION

The "stem" is the tens digit and the "leaves" are the units digits. This list has numbers below 10, in the teens, the 20s, the 30s, the 40s, and 100s. Put the tens in ascending order in the stem column, then go through each number and put the "leaf" in the appropriate column as in Table 1. Next, rearrange the leaves in ascending order as in Table 2.

TABLE 1	
Stem	*Leaf*
0	3
1	6, 8, 9, 2
2	3, 6, 8, 5, 4, 0
3	6, 3, 2, 4, 5
4	5, 3, 8, 5, 0
10	5

TABLE 2	
Stem	*Leaf*
0	3
1	2, 6, 8, 9
2	0, 3, 4, 5, 6, 8
3	2, 3, 4, 5, 6
4	0, 3, 5, 5, 8
10	5

Note: There are two 45s in the list, and they must both be listed because they are both part of the data.

Once you have completed the second table, write the numbers in ascending order.

3, 12, 16, 18, 19, 20, 23, 24, 25, 26, 28, 32, 33, 34, 35, 36, 40, 43, 45, 45, 48, 105

BUILD YOUR SKILLS

1. Write the following numbers in descending order using a stem and leaf plot.

   112, 129, 149, 155, 142, 123, 122, 115, 145, 144, 113

2. Organize the heights of the players on a basketball team from shortest to tallest using a stem and leaf plot.

   6′2″, 6′5″, 5′9″, 6′8″, 6′7″, 5′11.5″, 6′9″, 6′4″, 7′0.5″, 6′7.5″, 5′11″, 6′4.5″, 7′2″, 7′

NEW SKILLS: WORKING WITH ARITHMETIC MEAN

A **measure of central tendency** is a number around which a set of data clusters. In mathematics, there are several different measures of central tendency. The most common is the **arithmetic mean**, usually called an average in everyday language.

The arithmetic mean is denoted by the symbol $\bar{x}$. It is found by adding all the values in the data set and dividing this sum by the number of values in the set.

For more details, see page 110 of *MathWorks 12*.

There are other kinds of means, such as the geometric mean and the harmonic mean. In this course, you will only learn about the arithmetic mean.

measure of central tendency: a measure of the midpoint of an ordered data set

arithmetic mean: the sum of a set of values divided by the number of values; a measure of central tendency

Example 2

Raina's soccer team scored the following number of goals in its last 10 games:

7, 2, 4, 7, 0, 1, 3, 2, 6, 1

What was the team's mean number of goals scored per game?

SOLUTION

To find the mean score, add the scores together and divide by the number of games.

$$\bar{x} = \frac{7+2+4+7+0+1+3+2+6+1}{10}$$

$$\bar{x} = \frac{33}{10}$$

$$\bar{x} = 3.3$$

ALTERNATE SOLUTION

When there are a lot of numbers in the data set, you can first calculate the sum before calculating the mean.

sum of scores = 7 + 2 + 4 + 7 + 0 + 1 + 3 + 2 + 6 + 1

sum of scores = 33

$$\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$$

$$\bar{x} = \frac{33}{10}$$

$$\bar{x} = 3.3$$

Raina's team's mean score was 3.3 goals per game.

BUILD YOUR SKILLS

3. Calculate the mean of the following data sets.

   a) 3, 7, 12, 8, 6, 0, 19

   b) 129, 167, 193, 112, 145, 111

   c) 12, 7, –2, 3, –4, 5, –7

4. Chi's basketball team scored 78, 95, 69, 101, and 81 points in its last 5 games. What was the team's mean score per game?

The Canadian Rangers are a sub-component of the Canadian Forces that patrol northern, coastal, and isolated areas of Canada. The Canadian Rangers have a training camp in Alert, Nunavut, where they develop patrolling and disaster planning skills as well as traditional skills learned from the local community.

5. The following table shows the average daily temperature, by month, in Alert, Nunavut, over the past 40 years.

AVERAGE DAILY TEMPERATURE, BY MONTH, IN ALERT, NUNAVUT (°C)											
Jan.	*Feb.*	*Mar.*	*Apr.*	*May*	*Jun.*	*Jul.*	*Aug.*	*Sep.*	*Oct.*	*Nov.*	*Dec.*
–32.4	–33.4	–32.4	–24.4	–11.8	–0.8	3.3	0.8	–9.2	–19.4	–26.4	–30.1

What is the mean annual temperature?

Example 3

In one game, a football team has offensive drives of 45 yd, 43 yd, 24 yd, 21 yd, and 44 yd. The coach wants the team to achieve a mean of 35 yd per drive for the game. What must the length of the next offensive drive be if the team wants to meet this goal?

SOLUTION

There are a total of 6 offensive drives, the 5 known distances and the one unknown. Let the length of the unknown drive be x.

Use the equation for calculating mean.

$$\bar{x} = \frac{\text{sum of drives}}{\text{number of drives}}$$

$$35 = \frac{45 + 43 + 24 + 21 + 44 + x}{6}$$

$$35 = \frac{177 + x}{6}$$

$$6 \times 35 = \frac{177 + x}{6} \times 6$$

$$210 = 177 + x$$

$$210 - 177 = x$$

$$33 \text{ yd} = x$$

The team's next offensive drive will have to be 33 yd to reach the desired average.

BUILD YOUR SKILLS

6. In one week, the mean number of passengers through a transit turnstile per day is 1290. The table below shows the number of people who passed through the turnstile per day.

NUMBER OF TRANSIT PASSENGERS PER DAY	
Day	*Number of passengers*
Monday	1475
Tuesday	1321
Wednesday	1089
Thursday	1142
Friday	1421
Saturday	1654
Sunday	?

a) How many people passed through the turnstile on Sunday?

b) Does this number seem logical? What might be a explanation for this number?

7. Fran scored 86%, 74%, 82%, and 83% on her first four assignments in French class. What score must she get on her next test if she wants a mean of 86%? Explain what this score means.

Different types of grasses require different cutting lengths. Warm-season grasses are cut to a height of $\frac{1}{2}$ to 1 inch and cool-season grasses are cut to 2 to 3 inches.

8. Sven mows lawns to earn extra money. He charges different amounts based on the size and difficulty of the lawn.

a) How much must Sven charge for his next job if he wants to earn a mean of $25.00/lawn and he has charged $10.00, $35.00, $15.00, $40.00 for the first four lawns?

b) How much must he charge if he also mowed his grandmother's lawn for free, and he still wants to average $25.00 per lawn?

NEW SKILLS: WORKING WITH MEDIAN AND MODE

Median and **mode** are two other measures of central tendency.

median: the midpoint of an ordered data set—half the values are higher and half are lower; a measure of central tendency

mode: the value that occurs most often in a set of data; a measure of central tendency

The median of a set of numbers is the middle value in the data set. It is most easily found when the numbers are listed in ascending or descending order. If there is an even number of values in the data set, the median is the average, or mean, of the two middle numbers.

The mode of a set of numbers is the number that occurs most often in the list. A set of data can have:

- no mode if each number appears the same number of times;
- one mode if one number occurs more than each of the others; or
- a number of modes if two or more numbers occur the same number of times, and this is the greatest number of times an entry occurs in the data set.

Although the mode is referred to as a measure of central tendency, it does not necessarily lie toward the centre of the list. The mode is used in different situations than the mean or the median. For example, a store owner could use the mode to find out which size of T-shirt is the most popular, and use the information to decide how many T-shirts of each size to order in the future. The mean and median wouldn't help the store owner make this decision.

For more details, see page 111 of *MathWorks 12*.

Example 4

Calculate the median and the mode of the following data sets.

a) 25, 23, 32, 45, 32, 38, 28, 41, 45, 33, 29, 42, 45, 22, 35

b) 11, 3, 16, 29, 31, 65, 79, 51

SOLUTION

a) Arrange the numbers in ascending (or descending) order. You can use the stem and leaf plot method.

Stem	*Leaf*
2	5, 3, 8, 9, 2
3	2, 2, 8, 3, 5
4	5, 1, 5, 2, 5

Stem	*Leaf*
2	2, 3, 5, 8, 9
3	2, 2, 3, 5, 8
4	1, 2, 5, 5, 5

In ascending order, the numbers are:

22, 23, 25, 28, 29, 32, 32, <u>33</u>, 35, 38, 41, 42, 45, 45, 45

There are 15 numbers, so the median is the 8th number in the list. The median is 33.

Examine the list to see which number occurs most often. There are three 45s in the list, so the mode is 45.

b) Arrange the numbers in ascending order.

3, 11, 16, <u>29</u>, <u>31</u>, 51, 65, 79

There are 8 numbers, so the median is the mean of the two middle numbers, 29 and 31.

$$\text{median} = \frac{29 + 31}{2}$$

$$\text{median} = \frac{60}{2}$$

$$\text{median} = 30$$

None of the numbers appears more than once in the list, so the data set has no mode.

BUILD YOUR SKILLS

9. Denise is trying to choose a new computer. She has looked at models with the following prices:

$755.00, $1272.00, $845.00, $755.00, $1388.00, $1682.00, $909.00

a) What is the median price?

b) What is the mode?

10. The maximum daily temperatures (in °C) in Emerson, MB, for a two-week period in July were:

    26, 32, 28, 26, 24, 25, 23, 23, 26, 24, 21, 20, 22, 21

    a) What is the median maximum daily temperature?

    b) What is the mode?

The hottest temperature ever recorded in Manitoba was 42.2°C on July 11, 1936, in Winnipeg.

Example 5

Alexis is a hair stylist in Victoria, BC. In one shift, she earned the following tips:

$5.00, $2.00, $0, $20.00, $19.65, $7.89, $8.50, $5.00

a) What is the mean tip Alexis received?

b) What is the median?

c) What is the mode?

d) Which measure of central tendency best represents the average tip Alexis received?

SOLUTION

a) Calculate the mean.

$$\bar{x} = \frac{\$5.00 + \$2.00 + \$0 + \$20.00 + \$19.65 + \$7.89 + \$8.50 + \$5.00}{8}$$

$$\bar{x} = \frac{\$68.04}{8}$$

$$\bar{x} \approx \$8.51$$

b) Calculate the median tip. Organize the tips from lowest to highest.

$0, $2.00, $5.00, <u>$5.00</u>, <u>$7.89</u>, $8.50, $19.65, $20.00

The median is the mean of the two middle values, $5.00 and $7.89.

$$\text{median} = \frac{\$5.00 + \$7.89}{2}$$

$$\text{median} = \frac{\$12.89}{2}$$

$$\text{median} \approx \$6.45$$

c) The value $5.00 occurs twice in the list and it is the only value that appears more than once. It is therefore the mode.

d) The mean is not a good measure of central tendency for this data set. The mean of $8.51 is higher than all but two of the values in the data set.

The median and the mode could both be good representations of Alexis's average tip.

BUILD YOUR SKILLS

11. Elliott is a real estate agent in Regina, SK. In the past six months, he has sold houses for the following prices:

$265 000.00, $259 900.00, $276 000.00, $301 000.00, $519 900.00, $295 000.00, $271 895.00, $219 305.00

a) Calculate the mean, median, and mode house prices.

b) Which measure of central tendency do you think best represents the average house price? Explain your answer.

12. A large grocery store sells 13 varieties of whole wheat bread. The prices per loaf are:

    \$2.59, \$5.99, \$3.49, \$3.99, \$4.99, \$3.49, \$3.99, \$5.49, \$5.99, \$4.29, \$4.69, \$4.99, \$5.09

    a) Calculate the mean, median, and mode prices.

    b) For each measure of central tendency, explain whether it is or is not a good indicator of the average bread price.

13. Chandra makes gourmet chocolates and sells them at markets and craft fairs. In one evening at a market, she sold packages of the following sizes:

    75 g, 100 g, 175 g, 250 g, 75 g, 100 g, 100 g, 175 g, 100 g, 75 g, 100 g, 100 g

    a) Calculate the mean, median, and mode package sizes.

    b) Chandra is deciding on how many packages of each size she should bring to the next market. Which measure of central tendency should she use to help her decide? Explain your answer.

Chocolate is made from cocoa beans. Cocoa beans are the seeds found inside the fruit of the cacao tree. The different colours represent the fruit pods in various stages of ripeness.

PRACTISE YOUR NEW SKILLS

1. Calculate the mean, median, and mode of the following data sets.

   a) 2, 8, 3, 5, 7, 6, 10, 3, 2, 9, 7, 8

   b) 6°C, 8°C, –3°C, –7°C, 5°C, –6°C, –3°C

2. The annual salaries for the employees of a landscaping business are:

   $32 250.00, $23 300.00, $19 380.00, $23 300.00, $18 540.00, $45 000.00

   a) What is the mean salary?

   b) What is the median salary?

   c) What is the mode?

3. Lytton, BC, often has the highest daily temperature of Canadian cities in the summer. The following table shows the maximum temperature per month over a 40-year period.

MAXIMUM MONTHLY TEMPERATURE, LYTTON, BC (°C)											
Jan.	*Feb.*	*Mar.*	*Apr.*	*May*	*Jun.*	*Jul.*	*Aug.*	*Sep.*	*Oct.*	*Nov.*	*Dec.*
15.0	16.4	23.7	33.1	40.4	39.1	42.2	41.5	38.7	29.9	20.0	17.8

a) What are the mean, median, and mode maximum temperatures?

b) What do these measures of central tendency tell you about the maximum temperature in Lytton? Do any of the measures provide useful information? Explain your answer.

4. Yassir's grades on four quizzes were 65%, 71%, 84%, and 75%. What grade must he get on the next test to get an average of 76%?

5. Athena is a grocery store manager. She is looking at the store's sales of different brands of orange juice. Which measure of central tendency should she use to determine the most popular brand of orange juice?

3.2 Weighted and Trimmed Means and Outliers

NEW SKILLS: WORKING WITH OUTLIERS AND TRIMMED MEANS

In some of the questions in the previous section, the calculated mean, median, and mode were quite different. These calculations could be skewed, or distorted, by the distribution of the data.

If there is a number that is quite different from the other numbers in the list, this number is an outlier. As discussed in section 1.3, an outlier is a data point that is significantly different from the general trend in the data set. An outlier can have a considerable effect of the arithmetic mean of the data. To account for this, a **trimmed mean** may be a more representative measure of central tendency. When there are outliers, a trimmed mean is found by omitting an equal number of values from the upper and lower ends of the set of data.

trimmed mean: the mean calculated by disregarding an equal number of data points from both ends of the range

For more details, see pages 122–123 of *MathWorks 12*.

Example 1

A data set contains the following values:

3, 9, 5, 8, 4, 6, 20

a) Calculate the arithmetic mean.

b) Identify any outliers and calculate the trimmed mean.

c) Explain the difference between the arithmetic mean and the trimmed mean.

SOLUTION

a) Calculate the arithmetic mean.

$$\bar{x} = \frac{3 + 9 + 5 + 8 + 4 + 6 + 20}{7}$$

$$\bar{x} = \frac{55}{7}$$

$$\bar{x} \approx 7.9$$

The arithmetic mean is 7.9.

b) To identify outliers, it is often helpful to organize the data in ascending order.

3, 4, 5, 6, 8, 9, 20

Looking at the organized list, it is easier to see that 20 is an outlier because it differs significantly from the other numbers.

To calculate the trimmed mean, omit the outlier, 20. A trimmed mean requires omitting an equal number of values at the upper and lower ends of the data set, so the lowest number, 3, will also have to be omitted.

Calculate the mean of the remaining 5 numbers.

$$\bar{x} = \frac{4 + 5 + 6 + 8 + 9}{5}$$

$$\bar{x} = \frac{32}{5}$$

$$\bar{x} = 6.4$$

The trimmed mean is 6.4.

ALTERNATIVE SOLUTION

Rather than re-adding the numbers to find the trimmed mean, you could take the total calculated in a), 55, and subtract the two omitted numbers. Remember to subtract the number of omitted data entries (in this case, 2) from the total number of data entries.

$$\bar{x} = \frac{55 - 3 - 20}{7 - 2}$$

$$\bar{x} = \frac{32}{5}$$

$$\bar{x} = 6.4$$

> The subtraction method works best if there are a lot of data points or if it is easy to calculate the sum of the omitted numbers.

c) The arithmetic mean (7.9) and the trimmed mean (6.4) are very different. The trimmed mean is considerably lower than the arithmetic mean because the number 20 raised the sum of data points in the calculation of the arithmetic mean. In the trimmed mean, although 3 was also omitted, it was not different enough from the other numbers in the group to counteract the effect of the 20 on the mean.

BUILD YOUR SKILLS

1. A data set contains the following numbers:

   125, 164, 183, 154, 132, 129, 524, 476, 165, 133

   a) Calculate the arithmetic mean.

   b) Identify any outliers if they exist and calculate the trimmed mean.

The chinook arch is a band of stationary clouds caused by air rippling over the mountains. It may look menacing, but chinook arches rarely produce rain or snow. Here, a chinook arch lingers above downtown Calgary.

2. Calgary, AB, is known for its chinook winds, warm winds that blow from the Pacific Ocean and that can raise the temperature significantly over a short period of time. The following table shows the daily high temperatures for one week in January, during which there was a chinook wind.

DAILY HIGH TEMPERATURES FOR ONE WEEK IN JANUARY, CALGARY, AB (°C)						
Sun.	*Mon.*	*Tues.*	*Wed.*	*Thurs.*	*Fri.*	*Sat.*
–12	–14	–5	15	–8	–10	–12

   a) What was the mean high temperature for the week?

   b) On what day did the chinook occur?

   c) What is the trimmed mean of the daily highs?

   d) Which is the better indicator of the expected daily high in Calgary for a week in January? Why?

3. The annual salaries for employees at Don's Plumbing Store are:

   $15 000.00, $26 000.00, $29 000.00, $27 000.00, $60 000.00, $30 000.00

   a) Calculate the mean salary.

   b) Identify any outliers and calculate the trimmed mean.

   c) Explain why there is a difference between the trimmed mean and the arithmetic mean.

NEW SKILLS: WORKING WITH WEIGHTED MEANS—PERCENTAGE OF A TOTAL

In a **weighted mean**, the data points are not all of equal value. For example, a teacher might assign different weights to different parts of a student's grade: assignments might be worth 45% of the final grade, a quiz worth 15%, and an exam worth the remaining 40%. In this case, each data point is worth a percentage of the total.

weighted mean: the mean calculated when certain data entries are worth more than others

For more details, see page 123 of *MathWorks 12*.

Example 2

Frieda earned grades of 85%, 72%, 65%, and 90% on four tests in one term in her math class.

a) If each test were worth the same percentage of her final grade, what would be her final grade?

b) What was her term mark if the first test was worth 10% of her final grade, the second and third were each worth 20%, and the fourth test was worth 50%?

SOLUTION

a) If all the tests were worth the same percentage of her final grade, her final grade could be calculated using the arithmetic mean.

$$\bar{x} = \frac{85 + 72 + 65 + 90}{4}$$

$$\bar{x} = \frac{312}{4}$$

$$\bar{x} = 78\%$$

Frieda's final grade would be 78%.

b) In this case, each test is worth a percentage of the final grade. This is a weighted mean. To find her final grade, you need to find 10% of her first grade, 20% of each of the next two, and 50% of the last and add the percentage values together.

$$\bar{x} = (85 \times 0.10) + (72 \times 0.20) + (65 \times 0.20) + (90 \times 0.50)$$

$$\bar{x} = 8.5 + 14.4 + 13 + 45$$

$$\bar{x} \approx 81\%$$

The weighted mean is 81%.

BUILD YOUR SKILLS

4. Carina is calculating her final grade in her Spanish class. In the course, assignments are worth 15% all together, each quiz is worth 5%, the presentation is worth 25%, and the final exam is worth 45%.

Carina earned:

- 8/10, 6.5/10, and 16/20 on her assignments
- 10/10, 7/10, and 7/10 on her quizzes
- 85/100 on her presentation
- 86% on her final exam

What is Carina's final grade?

5. Adi wrote three math tests each worth 20% of his final grade. He got marks of 75%, 80%, and 86%. What mark must he get on the final exam (worth 40%) if he wants a final grade of 86%?

6. Jeffrey sells computers and other equipment at an electronics store. He is compiling statistics on the spending habits of customers. The table below shows customers' spending (rounded to the nearest $100.00).

SPENDING (ROUNDED TO NEAREST $100.00), BY PERCENTAGE OF CUSTOMERS								
$100.00	*$200.00*	*$500.00*	*$800.00*	*$1000.00*	*$1200.00*	*$1300.00*	*$1500.00*	*$2000.00*
28%	38%	15%	7%	2%	4%	3%	2%	1%

What was the mean spending (rounded to the nearest dollar) per customer?

NEW SKILLS: WORKING WITH WEIGHTED MEANS—REPEATING ITEMS

The weighted mean calculation can be used in another situation, when there is repetition of entries in the data set. The end result is the same as the arithmetic mean, but the calculation may be easier.

In some countries such as Canada and the United States, tipping is customary and often expected. However, in countries such as China and Japan, tipping is not part of the culture.

Example 3

In her job as a server at a restaurant, Glenna earned 2 tips of $6.00, 3 tips of $8.00, 3 tips of $10.00, and 6 tips of $12.00.

Calculate the mean tip using two different methods.

SOLUTION

Method 1: The mean tip can be calculated using the arithmetic mean calculation.

sum of tips = $6.00 + $6.00 + $8.00 + $8.00 + $8.00 + $10.00 + $10.00 + $10.00 + $12.00 + $12.00 + $12.00 + $12.00 + $12.00 + $12.00

sum of tips = $138.00

$$\bar{x} = \frac{\$138.00}{14}$$

$$\bar{x} \approx \$9.86$$

Glenna's mean tip was $9.86.

Method 2: Calculate the tip as a weighted mean. Group the tips by value.

$$\bar{x} = \frac{(2 \times \$6.00) + (3 \times \$8.00) + (3 \times \$10.00) + (6 \times \$12.00)}{2 + 3 + 3 + 6}$$

$$\bar{x} = \frac{\$12.00 + \$24.00 + \$30.00 + \$72.00}{14}$$

$$\bar{x} = \frac{\$138.00}{14}$$

$$\bar{x} \approx \$9.86$$

Glenna's mean tip was $9.86.

BUILD YOUR SKILLS

7. Calculate the arithmetic mean of the following set of numbers:

   16, 16, 16, 18, 19, 19, 20, 20, 20, 20, 20, 25, 25, 25, 25, 32, 32, 35, 35, 35, 35

8. Josh wants to know how many sports students in his school practise. He did a survey and found that:

   - 25 students do 1 sport
   - 75 students do 2 sports
   - 32 students do 3 sports
   - 5 students do 4 sports

   What is the average number of sports practised by a student in Josh's school?

9. Luanne creates pottery that she sells at craft fairs around Saskatchewan. At one craft fair, she sold:

   - 5 items for $25.00 each
   - 12 items for $16.00 each
   - 3 items for $21.00 each
   - 6 items for $14.00 each
   - 1 item for $45.00

   What is the mean cost per item Luanne sold?

PRACTISE YOUR NEW SKILLS

1. Liviu is a real estate salesperson. He recently sold houses for the following prices:

   $1 356 000.00, $289 000.00, $359 000.00, $376 900.00, $454 900.00, $376 500.00, $425 800.00

   a) Calculate the mean selling price.

   b) Identify any outliers and calculate the trimmed mean.

   c) Which calculation do you think best represents the average price of a house sold by Liviu?

2. Ghazale earned 90% on her assignments in her geography class, 85% on the quizzes, and 88% on the tests. If the assignments are worth 10% of her final grade, the quizzes 40%, and the tests 50%, what will be Ghazale's final grade in geography?

A cross-section of a tree showing blue stain fungus. Blue stain fungus is spread between certain species of trees by the mountain pine beetle. The fungus blocks the tree from receiving nutrients, eventually causing the tree to die from starvation.

3. Isabelle is a woodworker in Hinton, AB. She specializes in making furniture out of blue pine—wood from trees killed by the mountain pine beetle. Isabelle records how long is takes her to produce each item, in order to help her set the selling price for the furniture. The last 10 items she has made have taken her the following times (in hours):

10, 12, 10, 14, 13, 9, 21, 10, 11, 15

a) What is the mean number of hours Isabelle spent on each piece of furniture?

b) Are there any outliers? If so, calculate the trimmed mean.

c) Which calculation do you think best represents the average number of hours spent on each piece of furniture, and why?

d) What might explain the outliers in the data set?

4. Todor works at a warehouse for a book publisher. He has to load 75 boxes of books on a truck.

- 25 of the boxes weigh 60 lb each
- 20 weigh 12 lb each
- 16 weigh 45 lb each
- the remainder weigh 30 lb each

What was the average weight per box?

Pallet jacks (shown above) and forklifts are essential when moving and loading heavy boxes.

5. The ticket prices for a festival are: $12.00 for children, $15.00 for youth, $25.00 for adults, and $18.00 for senior citizens. If 125 children, 241 youth, 543 adults, and 328 senior citizens attended, what was the average price paid per ticket?

3.3 Percentile Ranking

percentile: a number from 1 to 99 that shows what percentage of values in a data set are lower than the given value

percentile rank: the percentage of values in a data set that are lower than a given value

The term **percentile** has the same base as the term percent or percentage, so it relates to "out of 100." Percentiles are used to organize sets of data so that you can see how one value compares to other values in the data set. A **percentile rank** indicates the percentage of values of the given data that are *lower* than the given number. If a number is said to be at the 60th percentile, it means that 60% of the numbers in the set are *below* the given number.

A percentile rank indicates where a value stands with respect to the other values in the data set.

A percentile rank can be calculated using the following formula, where b is the number of values below the one being considered, and n is the total number of values in the data set:

$$PR = \frac{b}{n} \times 100$$

For more information, see pages 135–136 of *MathWorks 12*.

Example 1

A stadiometer is a piece of medical equipment used for measuring height. The one in the photo is a digital stadiometer.

The players on the senior boys' volleyball team have the following heights:

165 cm, 175 cm, 154 cm, 162 cm, 178 cm, 182 cm, 190 cm, 167 cm, 155 cm, 172 cm, 143 cm, 187 cm

Calculate the percentile rank of the players with the following heights:

a) 172 cm

b) 190 cm

SOLUTION

Write the numbers in order from lowest to highest:

143, 154, 155, 162, 165, 167, 172, 175, 178, 182, 187, 190

a) Use the formula for calculating percentile rank. There are 12 heights in the set, and 6 of them are below 172 cm.

Percentile ranks are rounded to the nearest whole number.

$$PR = \frac{b}{n} \times 100$$

$$PR = \frac{6}{12} \times 100$$

$$PR = 50$$

The player with a height of 172 cm is in the 50th percentile rank. This means that 50% of the boys are shorter than 172 cm.

b) There are still 12 heights in the data set, and 11 of them are below 190 cm.

$$PR = \frac{b}{n} \times 100$$

$$PR = \frac{11}{12} \times 100$$

$$PR \approx 92$$

The player with a height of 190 cm is in the 92nd percentile. This means that 92% of the team is shorter than 190 cm.

Note that the highest percentile rank possible in a data set is 99. A percentile ranking tells you what percentage of values are lower than the value being ranked, and it is impossible for 100% of the values in a list to be lower than the given value.

BUILD YOUR SKILLS

1. In the following data sets, calculate the percentile rank of the number 8.

a) 6, 8, 12, 25, 30, 75, 80, 130, 230, 260

b) 7, 8, 9, 10, 11, 12, 13, 14, 15, 16

2. The students in a drama class earned the following grades on a presentation:

   75%, 76%, 77%, 79%, 80%, 81%, 82%, 84%, 85%, 87%, 88%, 89%, 90%, 91%, 92%, 94%, 95%, 96%, 98%, 100%

   a) Jordie earned a grade of 80%. What was his percentile rank?

   b) Meg earned 75%. What was her percentile rank?

   c) Jas earned 100%. What was his percentile rank?

The average newborn baby weighs between 6 and 9 pounds (2700 to 4000 grams).

3. There were 14 babies born at the St. Boniface Hospital in Manitoba in one day. The babies' weights, in grams, were:

   1540, 2230, 2720, 2780, 2890, 2910, 2930, 3050, 3140, 3190, 3650, 3890, 4210, 4420

   a) Monique's baby weighed 3050 g. What was the baby's percentile rank for weight?

   b) Mireille's baby weighed 2230 g. What was the baby's percentile rank for weight?

   c) What was the percentile rank for weight of the heaviest baby?

Example 2

Tom and Sarjit have just completed the Mortgage Brokerage in British Columbia course in order to become mortgage brokers. There were 175 people in the course.

a) If 90 of the students scored lower than Tom in the course, what was his percentile rank?

b) If 50 of the students scored higher than Sarjit and 3 people got the same score as him, what was his percentile rank?

c) What can you say about how Tom and Sarjit's scores compare to each other?

SOLUTION

a) Use the formula for calculating percentile rank.

$$PR = \frac{b}{n} \times 100$$

$$PR = \frac{90}{175} \times 100$$

$$PR \approx 51$$

Tom's percentile rank is 51. This means that 51% of the people scored lower than he did in the course.

b) Calculate how many people scored *lower* than Sarjit. There were 50 people that scored higher, plus 3 that scored the same, plus Sarjit himself—subtract these numbers from the total number of students.

$$175 - 50 - 3 - 1 = 121$$

121 people scored lower than Sarjit.

$$PR = \frac{b}{n} \times 100$$

$$PR = \frac{121}{175} \times 100$$

$$PR \approx 69$$

Sarjit's score was in the 69th percentile.

c) Tom's percentile rank of 51 and Sarjit's of 69 means that Sarjit did better than Tom, but that is all that you can say about their comparative scores. It is possible that their grades differed by very little or by a lot; the percentile ranking does not give you this information.

BUILD YOUR SKILLS

4. If 2000 people wrote a college entrance exam, and 385 scored lower than Peter, what was Peter's percentile rank?

Selling a home can be very complicated. Many home sellers hire a real estate agent to help guide them through the process.

5. Boris is selling his house in Saskatoon, SK, to move to Maple Creek, SK, to start a new job as a nurse. If 250 houses were sold in Saskatoon, 150 cost more than Boris's house, and none were the same price as his house, what was the percentile rank for the selling price of Boris's house?

6. An outdoor sporting goods store in Fort McMurray, AB, sells gloves of different sizes. The table below indicates the number of pairs of each size sold in the past month.

GLOVES SOLD IN ONE MONTH						
Size	*6.5*	*7*	*7.5*	*8*	*8.5*	*9*
Number of pairs	10	25	57	32	21	8

a) Dawnette bought herself size 7.5 gloves. What is the percentile rank of her hand size?

b) Dawnette bought her son size 9 gloves. What is the percentile rank of her son's hand size?

c) Explain why a percentile rank can never be 100 or greater.

Example 3

John participated in a survey of people's wages in his town. He learned that his wage was in the 70th percentile, and that 180 people were surveyed.

a) How many people surveyed earn less than John?

b) How many people earn more than John?

SOLUTION

a) Use the formula for calculating percentile rank, and solve for ***b***.

$$PR = \frac{b}{n} \times 100$$

$$70 = \frac{b}{180} \times 100$$

$$70 \div 100 = \frac{b}{180} \times 100 \div$$

$$0.7 = \frac{b}{180}$$

$$180 \times 0.7 = \frac{b}{180} \times 180$$

$$180 \times 0.7 = b$$

$$126 = b$$

Of the people surveyed, 126 earn less than John.

b) Since 126 earn less than John, you might expect to calculate the number of people who earn more as follows (with the –1 being for John's earnings):

$180 - 126 - 1 = 53$

However, you do not have enough information to calculate how many people earn more than John, because some of those 53 people might earn the same amount as John.

BUILD YOUR SKILLS

7. Petra has just completed a diploma from Nunavut Arctic College to become an environmental technologist. She has been checking job postings for work in her field. She has found the following statistics, showing earnings (rounded to the nearest $5000.00) for environmental technologists in her region.

ANNUAL SALARIES FOR ENVIRONMENTAL TECHNOLOGISTS				
Salary	*$25 000.00*	*$30 000.00*	*$35 000.00*	*$40 000.00*
Number of employees	6	9	11	3

If Petra gets a job that puts her in the 21st percentile in terms of salary, what is her salary?

8. Lena is a salesperson at an electronics store. The employees' sales are recorded to track their sales performance. There are 90 employees in the store, and Lena's sales place her in the 80th percentile. How many people made fewer sales than Lena?

9. A life insurance company bases its rates on the age at which a person applies for insurance. The table below indicates the number of people in different age groups who bought their life insurance from the company last year.

LIFE INSURANCE SALES, BY AGE GROUP					
Age group (years)	20–29	30–39	40–49	50–59	60–69
Number of buyers	123	231	102	51	5

a) What is the percentile ranking of a 30-year-old insurance buyer?

b) What is the percentile ranking of a 60-year-old insurance buyer?

c) What do you think the percentile ranking of a 45-year-old would be? Explain your answer.

PRACTISE YOUR NEW SKILLS

1. Marcel has a small organic vegetable farm, and he sells his produce at markets around Vancouver. He keeps track of the weights of tomatoes grown on some of his new plants, to see how well they are producing. He sells tomatoes with the following weights in grams:

   115, 111, 125, 160, 135, 133, 121, 101, 99, 125, 150, 145, 113

   a) What is the percentile rank of a tomato weighing 135 g?

   b) What is the percentile rank for a tomato weighing 125 g?

The Vancouver Farmers' Market at Trout Lake is a popular place on Saturdays from May to October. The market features a variety of foods such as fruits and vegetables, artisanal cheeses, and baked goods.

2. There are 120 students in grade 10 at a high school in Whitehorse, YT. They took an English exam.

   a) Of the 120 students, 16 earned a lower mark than Joe. What was his percentile rank?

   b) If Marina's percentile rank was 85, how many students earned a lower mark than she did?

   c) Two students earned the same mark as Dan, and 45 students earned a higher mark than Dan. What was Dan's percentile rank?

HOUSE PRICES IN WINNIPEG, MB	
Price range ($)	*Number of houses*
0–99 999	6
100 000–199 999	19
200 000–299 999	36
300 000–399 999	17
400 000–499 999	5
500 000–599 999	10
600 000–699 999	1
700 000–799 999	2
800 000–899 999	1
900 000–999 999	2
1 000 000+	3

3. The table shows a random sampling of 102 properties listed for sale in Winnipeg, MB.

   a) What is the percentile rank of a house listed in the $300 000–$399 999 range?

   b) What is the percentile rank of a property that sold for $1 000 000 or more?

   c) A house in the 81st percentile was sold. In what price range was the house?

CHAPTER TEST

1. Albert scored 66%, 89%, 73%, 73%, 89%, and 72% on assignments in his math class. The assignments are worth 40% of his final grade, and the final exam is worth 60%. If he earns a grade of 75% on his final exam, what will be his final mark in the course?

2. For a school production, Marcia bought the fabric for making costumes and backdrops. The different fabrics cost varying amounts per metre. Calculate the mean cost per metre of cloth.

COST OF FABRIC BOUGHT FOR SCHOOL PRODUCTION			
Cost per metre	$5.89	$12.49	$35.29
Number of metres purchased	38	12	3

One of the most expensive cloths in the world is made from the wool of the vicuña, a wild ancestor of the domesticated alpaca, found in South America. The wool is so rare because vicuña only produce 1 lb of wool a year. Vicuña yarns and fabrics can range from $1800.00 to $3000.00 a yard.

3. Dev is studying in the apprenticeship plumber/gasfitter program at Aurora College's Thebacha Campus in Fort Smith, NT. In his blueprint reading, sketching, and drawing course, he has earned the following marks:

- 70% on the first assignment
- 79% on the second assignment
- 68% on the first quiz
- 81% on the second quiz

To calculate his final grade, each assignment is worth 10%, each quiz is worth 15%, and the final exam is worth 50%. If he wants to earn 80% in the course, what mark does he have to earn on the final exam?

4. The students in Madison's class have the following heights:

167 cm, 200 cm, 155 cm, 135 cm, 142 cm, 143 cm, 137 cm, 155 cm, 143 cm, 164 cm, 161 cm, 139 cm, 152 cm, 143 cm

a) Calculate the mean, median, and mode. Which do you think best represents the average height in the class? Why?

b) Identify any outliers in the data and calculate the trimmed mean.

c) What is the percentile rank of the student who is 152 cm tall?

5. The table below shows the time it takes students to get to school.

TRAVEL TIMES TO SCHOOL						
Travel time (minutes)	<10	10–14	15–19	20–24	25–29	>30
Number of students	15	25	16	31	47	12

a) What is the percentile rank for time for a person who takes more than 30 minutes to travel to school?

b) How long does it take a student in the 60th percentile to get to school?

Chapter 4

Probability and Odds

The likelihood of winning a board game, card game, or other contest can be calculated as a probability.

4.1 Experimental Probability

REVIEW: SIMPLIFYING FRACTIONS

In this section, you will need to simplify fractions to their lowest terms.

Example 1

Simplify these fractions to their lowest terms.

a) $\frac{39}{117}$ b) $\frac{32}{48}$

SOLUTION

To simplify a fraction, divide the numerator and the denominator by the largest common factor.

a) $\frac{39}{117} = \frac{39 \div 39}{117 \div 39}$

$\frac{39 \div 39}{117 \div 39} = \frac{1}{3}$

b) $\frac{32}{48} = \frac{32 \div 16}{48 \div 16}$

$\frac{32 \div 16}{48 \div 16} = \frac{2}{3}$

ALTERNATIVE SOLUTION

If you do not know what the largest common factor is, you can use any common factor and simplify in several steps.

a) $\frac{39}{117} = \frac{39 \div 3}{117 \div 3}$

$\frac{39 \div 3}{117 \div 3} = \frac{13}{39}$

$\frac{13}{39} = \frac{13 \div 13}{39 \div 13}$

$\frac{13 \div 13}{39 \div 13} = \frac{1}{3}$

b) $\frac{32}{48} = \frac{32 \div 2}{48 \div 2}$

$\frac{32 \div 2}{48 \div 2} = \frac{16}{24}$

$\frac{16}{24} = \frac{16 \div 4}{24 \div 4}$

$\frac{16 \div 4}{24 \div 4} = \frac{4}{6}$

$\frac{4}{6} = \frac{4 \div 2}{6 \div 2}$

$\frac{4 \div 2}{6 \div 2} = \frac{2}{3}$

BUILD YOUR SKILLS

1. Simplify these fractions to their lowest terms.

a) $\frac{45}{72}$

b) $\frac{168}{294}$

c) $\frac{675}{1575}$

d) $\frac{54}{81}$

REVIEW: WORKING WITH FRACTIONS, DECIMALS, AND PERCENTAGES

In this section, you will need to convert between fractions, decimals, and percentages.

Example 2

Complete the following table showing equivalent value for the fractions, decimals, and percentages.

CONVERTING BETWEEN FRACTIONS, DECIMALS, AND PERCENTAGES		
Fraction	*Decimal*	*Percentage*
$\frac{1}{5}$		
	0.70	
		12%

SOLUTION

To convert a fraction to a decimal, divide the numerator by the denominator.

To convert a decimal to a percentage, multiply by 100.

To convert a percentage to a fraction, put the number over 100 and simplify to the lowest terms.

CONVERTING BETWEEN FRACTIONS, DECIMALS, AND PERCENTAGES		
Fraction	*Decimal*	*Percentage*
$\frac{1}{5}$	$1 \div 5 = 0.2$	$0.2 \times 100 = 20\%$
$70\% = \frac{70}{100}$ $\frac{70}{100} = \frac{70 \div 10}{100 \div 10}$ $\frac{70 \div 10}{100 \div 10} = \frac{7}{10}$	0.70	$0.70 \times 100 = 70\%$
$12\% = \frac{12}{100}$ $\frac{12}{100} = \frac{12 \div 4}{100 \div 4}$ $\frac{12 \div 4}{100 \div 4} = \frac{3}{25}$	$3 \div 25 = 0.12$	12%

BUILD YOUR SKILLS

2. Complete the following table.

CONVERTING BETWEEN FRACTIONS, DECIMALS, AND PERCENTAGES		
Fraction	*Decimal*	*Percentage*
	0.89	
		67%
$\frac{2}{37}$		
	0.03	
		45%
$\frac{11}{12}$		

NEW SKILLS: WORKING WITH EXPERIMENTAL PROBABILITY

You may have heard expressions like this:

- Eight out of ten veterinarians recommend a certain brand of pet food.
- There is a 70% chance of rain today.

probability: the chance of an event happening, expressed as a percentage, fraction, decimal, or in words

experimental probability: a ratio of the number of occurrences of a particular event during an experiment or trial to the total number of trials conducted

These are expressions of **probability**. Probability is a prediction of how likely it is that a particular event will happen. Probability can be expressed as a fraction, a decimal, a percentage, or in words. The formula for calculating the probability (P) of an event *A* is:

$$P(A) = \frac{\text{number of occurrences of event } A}{\text{total number of possible outcomes}}$$

Probability is expressed in the lowest terms.

Experimental probability is calculated based on measuring occurrences of *A* in a sample data set or running a trial to find out how many times *A* occurs. Using the results of the sample or trial can allow you to make predictions about the likelihood of future occurrences.

For more details, see page 148 of *MathWorks 12*.

Example 3

Georgina bought a bag of bulk candies. She counted them and found that 15 were red, 10 were green, 12 were purple, and 13 were orange.

a) Calculate the probability of a randomly chosen candy from the bag being red. Express the probability as a fraction, a decimal, a percentage, and in words.

b) What is the probability of a randomly chosen candy being green or purple?

c) If the distribution of colours in a bag of 1000 candies is the same, how many would Georgina expect to be red?

To calculate the probability of choosing a red jelly bean from this jar, you would need to know how many jelly beans it contains. How many would you guess?

SOLUTION

a) The event being considered is the occurrence of red candies, which is 15. Use the formula for calculating probability.

$$P(\text{red candy}) = \frac{\text{number of red candies}}{\text{total candies}}$$

$$P(\text{red candy}) = \frac{15}{15 + 10 + 12 + 13}$$

$$P(\text{red candy}) = \frac{15}{50}$$

$$P(\text{red candy}) = \frac{15 \div 5}{50 \div 5}$$

$$P(\text{red candy}) = \frac{3}{10}$$

The probability of a randomly chosen candy from the bag being red is $\frac{3}{10}$. This can also be expressed as 0.3, 30%, or 3 out of 10.

b) The event being considered is the occurrence of green or purple candies.

$$P(\text{green or purple}) = \frac{\text{number of green or purple candies}}{\text{total candies}}$$

$$P(\text{green or purple}) = \frac{10 + 12}{50}$$

$$P(\text{green or purple}) = \frac{22}{50}$$

$$P(\text{green or purple}) = \frac{11}{25}$$

The probability of choosing a green or purple candy is $\frac{11}{25}$. This can also be expressed as 0.44, 44%, or 11 out of 25.

The probability of an event, when expressed as a decimal, is always a number from 0 and 1. If the probability is 0, the event is impossible. If the probability is 1, the event is certain.

c) You can calculate the number of candies likely to be red in the bag by multiplying the total number of candies in the bag by the probability of a candy being red.

$1000 \times 0.3 = 300$

In a bag of 1000 candies, 300 are likely to be red.

ALTERNATIVE SOLUTION

You can use proportional reasoning to solve for the number of red candies.

$$\frac{\text{number of red candies in small bag}}{\text{total number of candies in small bag}} = \frac{\text{number of red candies in large bag}}{\text{total number of candies in large bag}}$$

$$\frac{3}{10} = \frac{x}{1000}$$

$$1000 \times \frac{3}{10} = \frac{x}{1000} \times 1000$$

$$300 = x$$

BUILD YOUR SKILLS

3. A grocery store sells three brands of toothpaste. Last month, the store sold 75 tubes of brand A, 23 tubes of brand B, and 57 tubes of brand C.

   a) What is the probability of a customer buying a tube of brand A? Express your answer as a simplified fraction.

   b) If the store sells 500 tubes of toothpaste the next month, how many tubes of brand B can they expect to sell?

4. The table below shows the number of people of each blood type who donated blood at a recent drive.

BLOOD DONORS, BY TYPE								
Blood type	*O+*	*A+*	*B+*	*AB+*	*O–*	*A–*	*B–*	*AB–*
Number of donors	327	277	77	27	69	56	16	1

a) What is the probability that a person selected at random has the following blood type?

i) B+ ii) O iii) AB–

b) If the data approximately represent the blood type distribution of the general population, how many people in a group of 5000 are likely to have AB+ blood?

Mobile blood donation centres are used for blood drives at schools, recreation centres, and other locations in the community. The blood donations are used for transfusions during surgeries and for other medical treatments.

5. Brianne polled the students in the English 12 classes in her school to find out what types of books they prefer to read.

BOOK TYPE PREFERENCES						
Type of book	*Action*	*Science fiction*	*Poetry*	*Mystery*	*Historical novel*	*Math/Science*
Number of students	22	28	5	38	4	7

a) What is the probability that a randomly chosen student will like poetry books?

b) What is the probability that a randomly chosen student will like historical novels?

c) If the school is adding to the library and has funding to purchase 1000 books, how many should be science fiction?

NEW SKILLS: WORKING WITH SAMPLE SIZE

When performing an experiment to calculate experimental probability, you need to consider the sample size—that is, the total number of objects/events being counted. A very small sample size may not give results that are representative of the whole population. When the sample size is appropriate, the probability of the event of interest will be the same in the sample size as in the population as a whole.

Example 4

Cafeteria-style service is popular at many Filipino restaurants. In Tagalog, this style of eatery is called "turo-turo" or "point-point" to describe the action of pointing to the dish you want to order.

A large cafeteria that serves lunch to more than 400 people per day offers four main dishes: spaghetti, a hamburger, chicken chow mein, and a vegetarian dish. Of the first 10 people through the line, no one chose the vegetarian dish. The manager decided to remove vegetarian dishes from the menu in the future. Is this a good decision? Explain your answer.

SOLUTION

This is not a good decision, because there are likely to be some vegetarians who want to eat at the cafeteria but were not at the front of the line. The sample size of 10 people is very small compared to the population of 400. Rather than eliminating the vegetarian dish altogether, the manager should keep track of how many of each dish are sold and plan the amounts accordingly.

BUILD YOUR SKILLS

6. Katcha lives in Vancouver, BC. She conducted a survey in her math class and found that 29 out of the 30 students had brown eyes. She concluded that approximately 97% of the population of Canada has brown eyes. Is this a logical conclusion? Explain your answer.

7. Elections for city mayor are being held, and a polling company is conducting a telephone poll to predict the winner. The city has a population of 390 000. Of 100 people called, 78 say they will vote for Ms. Quon. Is it safe to say that Ms. Quon will win the election? Explain your answer.

8. A polling company did a survey to find out how much time people spend on the internet each week. The survey was conducted by telephone to home telephone numbers. The results showed that people spend on average 5 hours per week on the internet.

   A second company did a survey asking the same question, but asked the question in person rather than over the telephone. They conducted the survey at a shopping mall, a community centre, and a university campus. The results showed that people spend on average 15 hours per week on the internet.

   Give a possible reason for the discrepancy in the survey results.

PRACTISE YOUR NEW SKILLS

1. Complete the following table showing the alternative ways of expressing a probability.

WAYS OF EXPRESSING PROBABILITY			
Fraction	*Decimal*	*Percentage*	*Words*
$\frac{3}{4}$			
	0.058		
			24 out of 52
		74.5%	
$\frac{17}{55}$			

2. If 75 out of 87 beads in a box are white and the rest are multicoloured, what is the probability of choosing a multicoloured bead from the box? Write the answer as a fraction, a decimal, and as a percentage.

3. A sports equipment manufacturing company is producing basketballs. When 276 basketballs were tested, 3 were found to be defective.

The International Basketball Federation has strict specifications for basketballs used in competition. The balls must have a circumference of 749–780 mm for men's games and 724–737 mm for women's games, and must bounce to a height of 1200–1400 mm when dropped from a height of 1800 mm onto the playing surface.

a) What is the probability that a basketball chosen at random will be defective? Express the answer as a fraction.

b) If a store buys 5000 basketballs, how many are likely to be defective?

4. Mary flipped a coin 30 times. The coin landed on heads 12 times and on tails 18 times.

a) What is the experimental probability that this coin will land on tails?

b) Based on these experimental results, how many times would Mary expect the coin to land on heads in 600 coin tosses?

5. A teacher began grading a test. The first 3 papers all received a grade of 95%, so he concluded that the test was too easy. Is this a reasonable conclusion? Why or why not?

6. Elections for student council are being held at a high school of 512 students. A poll was taken to find out who students planned to vote for as president. The results of the poll are shown in the table.

STUDENT COUNCIL POLL RESULTS—VOTES FOR PRESIDENT			
Chloe	*Huan*	*Noah*	*Zahara*
15	43	9	35

a) What percentage of the student population was polled?

In the 2011 federal election, 61.1% of registered voters cast a ballot.

b) If this poll is representative of the whole student population and all students in the school cast a vote, how many votes can Noah expect to receive in the election?

c) How many votes can be expected to separate Huan and Zahara?

4.2 Theoretical Probability

theoretical probability: the ratio of one or more events of interest to the total number of possible outcomes

In the previous section, you calculated probability based on data from samples or trials. There is another type of probability calculation, called **theoretical probability**. It is the probability that an event will occur based on the number of ***possible*** occurrences, rather than the total number of items/events in a trial. For example, if you toss a coin, there are two possible outcomes: landing on tails or landing on heads. The theoretical probability of landing on heads is therefore 50%.

Compare this to the problem in the previous section (p. 152) where you calculated the experimental probability of a coin landing on tails based on a trial of 30 coin tosses. The answer was not 50%, because the coin landed on tails more times than it landed on heads. Theoretical and experimental probability may differ if the experimental sample size is smaller; however, in a large sample, the experimental probability should be close to the theoretical probability.

For more details, see page 156 of *MathWorks 12*.

Example 1

A standard deck of 52 playing cards consists of 4 suits: clubs, diamonds, hearts, and spades.

Diamonds and hearts are red. Clubs and spades are black.

Each suit contains 13 cards: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, and king.

Jacks, queens, and kings are called face cards. There are 12 face cards in a deck.

If you draw a card at random from a standard deck of 52 playing cards, what is the probability that it will be one of the following cards?

a) a heart b) a black jack c) a card other than a diamond?

SOLUTION

a) There are 13 hearts in one deck of cards, and 52 cards in total.

$$P = \frac{\text{number of events of interest}}{\text{total possible outcomes}}$$

$$P(\text{heart}) = \frac{\text{number of hearts}}{\text{total number of cards}}$$

$$P(\text{heart}) = \frac{13}{52}$$

$$P(\text{heart}) = \frac{1}{4}$$

The probability that a card randomly chosen from a deck of cards will be a heart is $\frac{1}{4}$. This can also be expressed as 0.25, 25%, or 1 out of 4.

b) There are 4 jacks in a deck of cards, but only 2 of them are black (the jack of clubs and the jack of spades).

$$P(\text{black jack}) = \frac{\text{number of black jacks}}{\text{total number of cards}}$$

$$P(\text{black jack}) = \frac{2}{52}$$

$$P(\text{black jack}) = \frac{1}{26}$$

The probability that a card randomly chosen from a deck of cards will be a black jack is $\frac{1}{26}$, 0.038, 3.8%, or 1 out of 26.

c) The deck has 3 suits that are not diamonds, and each suit has 13 cards.

$3 \times 13 = 39$

There are 39 cards that are not diamonds.

$$P(\text{not diamond}) = \frac{\text{number of cards that are not diamonds}}{\text{total number of cards}}$$

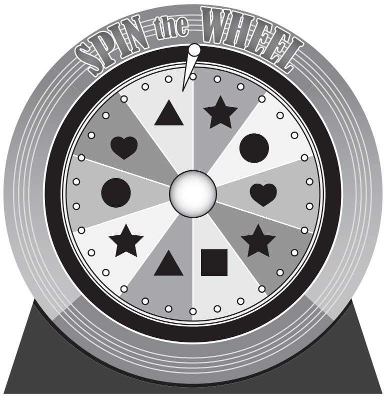
$$P(\text{not diamond}) = \frac{39}{52}$$

$$P(\text{not diamond}) = \frac{3}{4}$$

The probability that a card chosen randomly from a deck of cards will not be a diamond is $\frac{3}{4}$, 0.75, 75%, or 3 out of 4.

BUILD YOUR SKILLS

1. Use the spinner below to calculate the probabilities.



a) What is the probability that the spinner will stop on a star?

b) What is the probability that the spinner will not stop on a heart?

c) What is the probability that the spinner will stop on a square or a triangle?

2. A standard 6-sided die is tossed. What is the probability that it will land on:

a) 5?

b) an even number?

c) a multiple of 3?

3. A 12-sided die has faces numbered 1 to 12. If the die is tossed, what is the probability of it landing on:

a) an odd number?

b) a multiple of 2?

Dice come in many different shapes and sizes. The 12-sided die is a dodecahedron, and the 20-sided die is an icosahedron.

Example 2

a) What is the probability of rolling doubles with two standard six-sided dice?

b) Adele is playing a board game. If she rolls doubles on her first turn, what is the probability that she will roll doubles on her second turn?

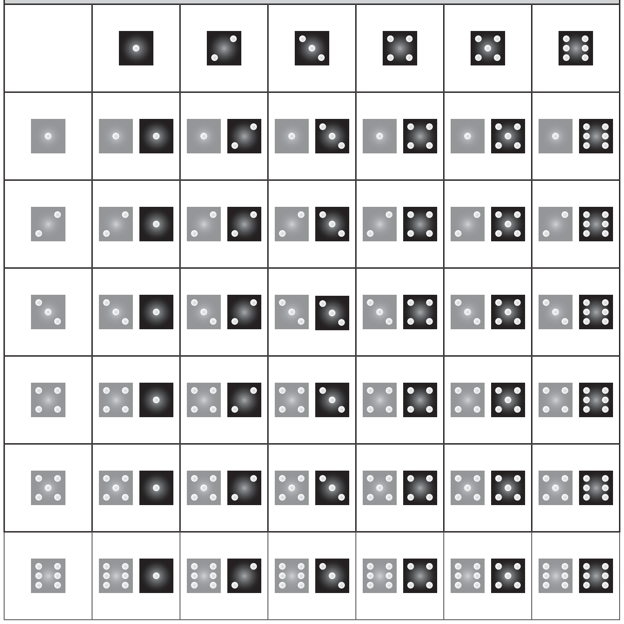
SOLUTION

a) There are 6 ways to roll doubles. Calculate how many possible outcomes there are when rolling two dice. There are 6 possible outcomes for the first die and 6 possible outcomes for the second die. Multiply the outcomes of each die to find the total possible outcomes.

6 possible outcomes for first die × 6 possible outcomes for second die = 36 possible outcomes for 2 dice

This can be illustrated in the following table, which shows all the possible outcomes when 2 dice are rolled.

DICE TABLE



Calculate the probability.

$$P(\text{doubles}) = \frac{\text{number of ways to roll doubles}}{\text{number of possible outcomes}}$$

$$P(\text{doubles}) = \frac{6}{36}$$

$$P(\text{doubles}) = \frac{1}{6}$$

The probability of rolling doubles is $\frac{1}{6}$, 0.17, 17%, or 1 out of 6.

b) This is not the same as the probability of rolling doubles twice in a row. The probability of rolling doubles a second time is not affected by the results of the first roll. There are still 6 ways to roll doubles, and 36 possible outcomes.

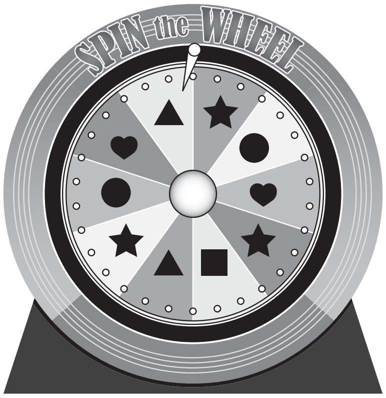
The probability of rolling doubles in Adele's second turn is still $\frac{1}{6}$, 0.17, 17%, or 1 out of 6.

BUILD YOUR SKILLS

4. Alfred rolled a die twice. The first time, it landed on 5. What is the probability that the second roll will land on 5?

5. A family consists of 3 children. If the first 2 are boys, what is the probability that the third is a boy?

6. Una used the spinner below, and it landed on a heart. If she spins it again, what is the probability that it will land on a heart?



Example 3

Louise is playing a card game. She draws one card from a standard deck of playing cards, then without replacing the card, draws a second card.

a) What is the probability that the first card drawn is an ace?

b) If the first card chosen was an ace, what is the probability that the second card is a 2?

SOLUTION

a) There are 4 aces in a deck of cards.

$$P(\text{ace}) = \frac{\text{number of aces}}{\text{total number of cards}}$$

$$P(\text{ace}) = \frac{4}{52}$$

$$P(\text{ace}) = \frac{1}{13}$$

The probability of Louise choosing an ace on her first draw is $\frac{1}{13}$, 0.077, 7.7%, or 1 out of 13.

b) On her second draw, the number of cards in the deck has changed. There are now 51 cards in the deck, and 4 of them are 2s.

$$P(2) = \frac{\text{number of 2s}}{\text{total number of cards}}$$

$$P(2) = \frac{4}{51}$$

The probability of choosing a 2 on her second draw is $\frac{4}{51}$, 0.078, 7.8%, or 4 out of 51.

BUILD YOUR SKILLS

7. Deshawn drew a card from a standard deck of playing cards. It was a face card. He did not put the card back in the deck. What is the probability that a second card drawn will be an ace?

After-school daycare programs often offer many activities such as crafts, games, and outdoor play.

8. Elizabeth works at an after-school daycare in Prince Albert, SK. She is helping some of the children make beaded necklaces. A bag contains 15 red, 12 blue, and 18 white beads.

   a) What is the probability that a randomly chosen bead will be white?

b) If the first bead drawn is white, and it is not replaced, what is the probability that the second bead drawn will be white?

c) If the first bead drawn is white, and it is not replaced, what is the probability that the second bead drawn will be blue?

d) From the original bag of beads (15 red, 12 blue, and 18 white), a white bead and blue bead were drawn and not replaced. What is the probability that a third bead drawn will be red?

PRACTISE YOUR NEW SKILLS

Which probability is higher: choosing a face card or choosing an even-numbered card?

1. If a card is randomly chosen from a standard deck of playing cards, what is the probability that it will be a:

a) queen?

b) red face card?

c) 2 or 3?

2. a) What is the theoretical probability of rolling a sum of 7 with two standard six-sided dice?

HINT

Use the table on page 157.

b) Pascale wanted to compare the theoretical probability to results of an experiment. She rolled 2 dice 50 times, and got a sum of 7 a total of 5 times. Calculate the experimental probability.

c) If the dice were rolled 1000 times, how many times do you think a sum of 7 would be rolled? Would you use theoretical or experimental probability? Explain your answer.

3. Madeleine is choosing fruit from a fruit bowl. The bowl contains 3 apples, 2 nectarines, and 1 peach.

a) If she randomly chooses a fruit without looking, what is the probability that she will take a peach?

b) Madeleine does take a peach, but decides she wants a different fruit. She places the peach back in the bowl, then randomly chooses again without looking. What is the probability that she will choose an apple?

4. A combination on a lock is made up of 3 numbers between 1 and 3.

   a) How many possible combinations are there?

   b) What is the probability that you will guess the correct combination on the first try?

5. Orrin is a teacher in Iqaluit, NU. He is using markers to illustrate a lesson on a white board. He has a box that contains 5 red markers, 4 blue markers, 6 green markers, and 4 black markers.

   a) What is the probability that the first marker he chooses will be blue?

   b) The first marker is blue, and Orrin does not replace it in the box. What is the probability that the next marker chosen will be red?

A 100-sided die was invented and manufactured in 1985 by Lou Zocchi.

6. A standard six-sided die has been rolled 5 times and has landed on 4 each time. What is the probability that the next roll will land on 5?

4.3 Odds and Probability

If someone says, "The odds of our team winning the hockey game are 3 to 1," what does this mean? **Odds** are similar to probability, but compare the number of desired events to the number of undesired events, or vice versa, rather than to the total number of events.

odds: a ratio that compares a desired event to the number of undesired events, or vice versa

For example, if in the past 4 games, the hockey team won 3 times, the experimental probability of winning is 3 out of 4. The odds in favour of winning, however, are a ratio of the wins to losses, or 3 to 1.

Odds can be written in favour or against a particular event. The hockey team's odds against winning is the ratio of losses to wins, or 1 to 3.

For more information, see page 167 of *MathWorks 12*.

Example 1

If you roll a standard six-sided die, what are the odds:

a) in favour of rolling a 6?

b) against rolling a 6?

SOLUTION

a) There is only one way to roll a 6, and there are 5 other possible outcomes.

odds in favour = favourable outcomes:unfavourable outcomes
odds in favour = 1:5

The odds in favour of rolling a 6 are 1:5, or 1 to 5.

b) Calculate the odds against rolling a 6.

odds against = unfavourable outcomes:favourable outcomes
odds against = 5:1

The odds against rolling a 6 are 5:1, or 5 to 1.

BUILD YOUR SKILLS

1. If a card is chosen randomly from a standard deck of playing cards, what are the odds:

   a) in favour of it being a diamond?

   b) against it being a queen?

Traditional incandescent light bulbs are being phased out of use in Canada, in favour of more energy-efficient bulbs such as compact fluorescent lamps (CFLs) and halogen lights.

2. A manufacturing plant makes light bulbs. As a quality control measure, Eli must test a random sample of light bulbs to check that they work. In a sample of 500 light bulbs, 3 were found to be defective.

   a) What are the odds in favour of a light bulb being defective?

   b) What are the odds against a light bulb being defective?

   c) What are the odds in favour of a light bulb working properly?

3. The odds in favour of choosing a red marble from a bag of marbles are 1:1. What does this mean about the numbers of marbles in the bag and their colours?

Example 2

Alexa has a small gardening business in Victoria, BC. She planted 14 daffodil bulbs in a client's garden in the fall. In the spring, 12 bulbs flowered.

a) Based on these statistics, what are the odds in favour of a bulb Alexa planted in another garden flowering?

b) What is the experimental probability that a planted bulb will flower?

Daffodils are perennial, meaning they live for several years. The bulbs should be planted in the fall, about 6 to 8 weeks before frost is expected. They flower in the spring.

SOLUTION

a) Use the results from the first client's garden to calculate the odds of a bulb flowering.

The number of favourable outcomes is 12 and the number of total possible outcomes is 14. Calculate the number of unfavourable outcomes.

$14 - 12 = 2$

odds in favour = favourable outcomes:unfavourable outcomes
odds in favour = 12:2
odds in favour = (12 ÷ 2):(2 ÷ 2)
odds in favour = 6:1

The odds in favour of another bulb flowering are 6:1.

b) There are 12 events of interest and 14 total possible outcomes.

$$P = \frac{\text{number of events of interest}}{\text{total possible outcomes}}$$

$$P = \frac{12}{14}$$

$$P = \frac{6}{7},\ 0.86, \text{ or } 86\%$$

The probability that a bulb planted in another garden will flower is $\frac{6}{7}$, 0.86, 86%, or 6 out of 7.

BUILD YOUR SKILLS

4. A grocery store is running a promotion: everyone who makes a purchase receives a ticket that may win them a prize.

   - 5000 tickets say: "Please play again."
   - 125 tickets say: "Save 5% on your next purchase of $50.00 or more."
   - 5 tickets say: "Save 50% on your next purchase under $100.00."

   a) What are the odds in favour of getting a ticket offering 50% off?

   b) What is the probability in favour of getting a ticket offering 50% off?

   c) What are the odds in favour of getting a "Please play again" ticket?

5. There are 12 green, 4 blue, 15 yellow, and 7 white marbles in a container.

   a) What are the odds in favour of a randomly chosen marble being white?

   b) What are the odds in favour of a randomly chosen marble being a colour other than yellow?

c) What is the probability that a randomly chosen marble will be a colour other than yellow?

6. Liam is a server for a catering company. At an evening event, he is serving a platter of mini quiches. His platter contains 20 ham and cheese quiches, 15 spinach quiches, and 10 crab quiches. If a guest randomly chooses a quiche from the plate:

   a) What are the odds in favour of taking a spinach quiche?

   b) What are the odds against choosing a crab quiche?

   c) What is the probability of choosing a ham and cheese quiche?

Quiche is a savory tart made with pastry crust and an egg-based filling. A popular variety is the quiche Lorraine, which originated in the Lorraine region of France. It is filled with bacon, cheese, and onion.

Example 3

An animal shelter is holding a raffle to raise money to build a new room on their existing building. They are offering a variety of prizes, including a free vet visit and bags of pet food. In total, 4600 tickets were sold. The odds in favour of winning a prize are 3:227.

a) What is the probability of winning a prize?

b) How many prize-winning tickets are there?

SOLUTION

a) Odds in favour of winning of 3:227 mean that for every 3 winning tickets, there are 227 that do not win, or that for every 230 tickets, 3 are winners.

$$P = \frac{\text{number of events of interest}}{\text{total possible outcomes}}$$

$$P = \frac{3}{230},\ 0.01, \text{ or } 1\%$$

The probability of winning a prize is $\frac{3}{230}$, 0.01, 1%, or 3 out of 230.

b) Use proportional reasoning. Let x be the number of prizes out of 4600 tickets. Odds in favour of winning of 3:227 mean that for every 230 tickets, 3 are winners.

$$\frac{3}{230} = \frac{x}{4600}$$

$$4600 \times \frac{3}{230} = \frac{x}{4600} \times 4600$$

$$\frac{4600 \times 3}{230} = x$$

$$\frac{13\,800}{230} = x$$

$$60 = x$$

There are 60 winning tickets.

BUILD YOUR SKILLS

7. The odds of winning a contest are approximately 5:1476. How many winners are there likely to be if there are 50 000 participants?

8. A quality control test at a car manufacturing plant found that the odds in favour of a windshield being installed correctly are 4999:1. If the plant assembles 84 025 cars in one year, how many are likely to have problems with the windshield installation?

9. Marie-Claude's softball team made it to the league finals. A sports reporter for the newspaper said that the odds in favour of the team winning the championship are 5:2. What is the probability that Marie-Claude's team will win?

Slow-pitch softball leagues are popular with people of all ages!

PRACTISE YOUR NEW SKILLS

1. Complete the following chart, showing the odds in favour, odds against, and probability of different events.

ODDS AND PROBABILITY			
Event	*Odds in favour*	*Odds against*	*Probability*
Rolling an even number with a six-sided die			
Rolling doubles with two dice			
Drawing a spade from a deck of cards			
Drawing a 5 from a deck of cards			

2. There are 8 men and 10 women in a room. Half the men and half the women have brown hair. If a person is randomly chosen from the room, what are the odds in favour of him/her having brown hair?

3. For a charity raffle, 1000 tickets are sold as a fundraiser. The odds against winning a prize are 199:1.

   a) How many winning tickets are sold?

b) What is the probability of winning a prize?

The first zipper-like device was invented by Elias Howe, the inventor of the sewing machine. In 1851, he patented an "automatic, continuous clothing closure."

4. A clothing manufacturer is testing the zippers that have been used in a popular style of jacket. Of 500 jacket zippers tested, 5 were defective.

a) What are the odds in favour of purchasing a jacket with a defective zipper?

b) What is the probability that a jacket bought from the plant will have a properly working zipper?

c) What are the odds against purchasing a jacket with a defective zipper?

CHAPTER TEST

1. Complete the following table showing the alternative ways of expressing a probability.

WAYS OF EXPRESSING PROBABILITY			
Fraction	*Decimal*	*Percentage*	*Words*
$\frac{3}{125}$			
	0.23		
			1 out of 16
		94%	

2. The playing pieces in a board game include a bag of 30 red chips, 20 blue chips, and 15 yellow chips.

   a) If Danica randomly chooses a chip from the bag, what is the probability that it will be yellow?

   b) What are the odds in favour of Danica choosing a yellow chip?

   c) Danica does draw a yellow chip, does not return it to the bag, then passes the bag to Ahmet. What are the odds in favour of Ahmet drawing a blue chip?

Board games have existed since ancient Egypt. Board games nowadays typically have a counter or game piece that moves along the playing board. A die or dice are rolled to move game pieces and provide an element of chance.

3. A toy factory produces model helicopters. In a quality control test of 675 helicopters, 8 were found to have broken propellers.

   a) What is the probability that a model helicopter will have a broken propeller?

   b) If the factory produces 100 000 model helicopters, how many can be expected to have broken propellers?

4. a) If a card is randomly chosen from a standard deck of playing cards, what is the theoretical probability that it will be an ace?

   b) Zach wanted to compare the theoretical probability to the results of an experiment. He shuffled the deck of cards, chose the top card, and recorded the result. He did this 50 times. In the 50 trials, he picked up an ace twice. Calculate the experimental probability of choosing an ace.

   c) How many times do you think an ace will be drawn in 2000 trials? Explain your answer.

5. A museum is holding a special promotion. The first 2000 patrons receive a coupon when they enter the museum. The odds in favour of winning a prize are 19:231.

   a) What is the probability of winning a prize if you are among the first 2000 patrons?

   b) How many prizes are there?

   c) If a class of 28 students visits the museum, how many students can be expected to win a prize?

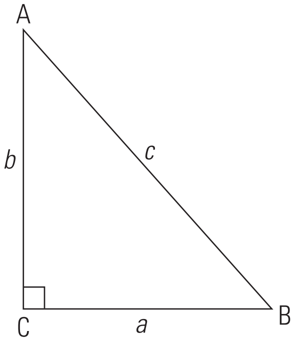
Chapter 5

Properties of Geometric Figures

The Lethbridge Viaduct, or more commonly known as the High Level Bridge, in Alberta, is the largest railway structure in Canada with a total length of 1624 m and a height of 95.7 m. The steel trestle was built between 1907 and 1909.

5.1 Triangles

REVIEW: WORKING WITH RIGHT TRIANGLES



In this section, you will need to use the Pythagorean theorem and trigonometric ratios when working with right triangles.

The Pythagorean theorem states that, for any right triangle:

$a^2 + b^2 = c^2$

where a and b are legs of the triangle and c is the hypotenuse.

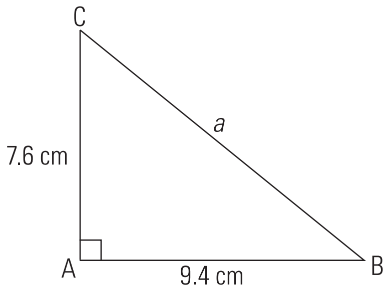
The primary trigonometric ratios are sine, cosine, and tangent. They are calculated as follows:

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

These ratios can be used to calculate missing angles or side lengths in right triangles.

Example 1

Calculate the length of the missing side.



SOLUTION

Use the Pythagorean theorem to solve for *a*.

$$b^2 + c^2 = a^2$$
$$(7.6)^2 + (9.4)^2 = a^2$$
$$57.76 + 88.36 = a^2$$
$$\sqrt{57.76 + 88.36} = a$$
$$\sqrt{146.12} = a$$
$$12.1 \text{ cm} \approx a$$

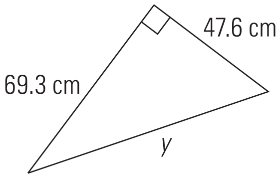
The hypotenuse is approximately 12.1 cm long.

Each vertex of a triangle is labelled with an upper case letter, and each side is labelled with a lower case letter corresponding to the opposite vertex. Side *a* is opposite ∠A.

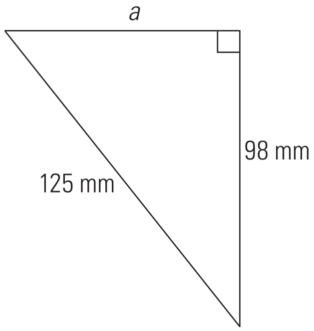
BUILD YOUR SKILLS

1. Use the Pythagorean theorem to find the missing side length.

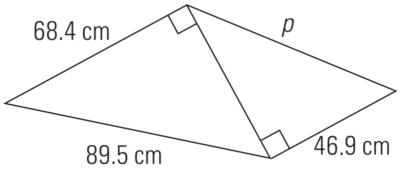
a)



b)

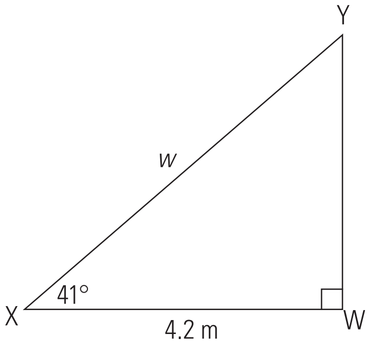


c)

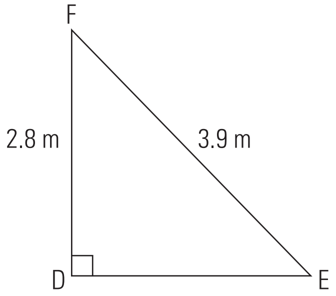


Example 2

a) Calculate the length of side *w*.



b) Calculate the measure of ∠E.



SOLUTION

a) Use the cosine ratio to solve for *w*.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 41° = \frac{4.2}{w}$$

$$w \times \cos 41° = \frac{4.2}{w} \times w$$

$$w \times \cos 41° = 4.2$$

$$\frac{w \times \cos 41°}{\cos 41°} = \frac{4.2}{\cos 41°}$$

$$w = \frac{4.2}{\cos 41°}$$

$$w \approx 5.6 \text{ m}$$

Side *w* is approximately 5.6 m.

b) Use the sine ratio to solve for ∠E.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin \text{E} = \frac{2.8}{3.9}$$

$$\text{E} = \sin^{-1}\left(\frac{2.8}{3.9}\right)$$

$$\text{E} \approx 46°$$

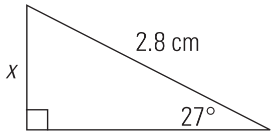
Angle E is approximately 46°.

BUILD YOUR SKILLS

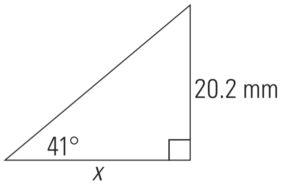
Roof trusses use right angles to create a rigid support for a roof.

2. Solve for the length of side x.

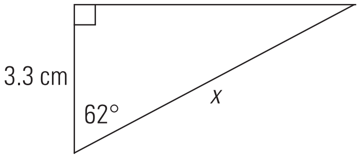
a)



b)

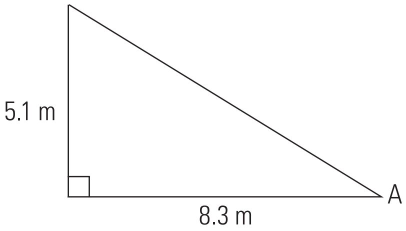


c)

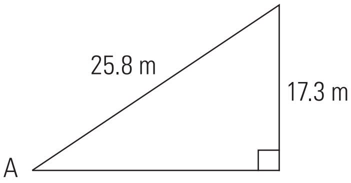


3. Solve for the measure of angle A.

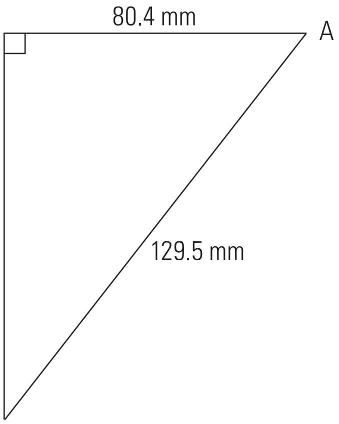
a)



b)



c)



NEW SKILLS: WORKING WITH PROPERTIES OF TRIANGLES

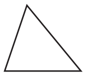
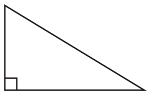
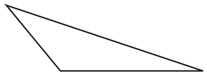
A property is a characteristic that classifies the object and distinguishes it among other objects of its type. What properties describe a **triangle**?

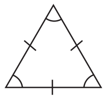
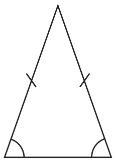
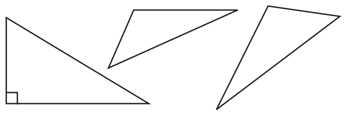
triangle: a polygon consisting of three line segments and three angles

- It has three sides;
- It is a closed figure; and
- The sum of interior angles is 180°.

For more details, see page 180 of *MathWorks 12*.

Triangles can be classified by their side lengths or by the sizes of the interior angles.

TRIANGLES CLASSIFIED BY ANGLE MEASURE		
acute	all three angle measures are less than 90°	
right	one angle measure is 90°	
obtuse	one angle measure is greater than 90°	

TRIANGLES CLASSIFIED BY SIDE LENGTH		
equilateral	three sides are of equal length and three angles are of equal measure	
isosceles	two sides are of equal length and two angles are of equal measure	
scalene	all sides are of different lengths and no angles are of equal measure	

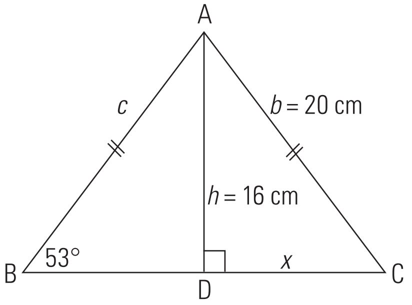
Triangles are useful in construction and manufacturing projects, because of their strength and rigidity. In this section, you will look at triangles in use in the real world.

For more details, see pages 180–181 of *MathWorks 12*.

Example 3

Triangles are important features of many structures, including Ferris wheels.

Use the following diagram to answer the questions below.



a) What is the length of side BC?

b) Classify ΔABC by angle measure and by side length.

SOLUTION

a) ΔABC can be divided into two right triangles. Use the Pythagorean theorem to calculate the length of DC.

$$x^2 + h^2 = b^2$$

$$x^2 + (16)^2 = (20)^2$$

$$x^2 + 256 = 400$$

$$x^2 = 400 - 256$$

$$x = \sqrt{400 - 256}$$

$$x = \sqrt{144}$$

$$x = 12 \text{ cm}$$

BC is twice the length of DC.

2×12 cm = 24 cm

Side BC is 24 cm long.

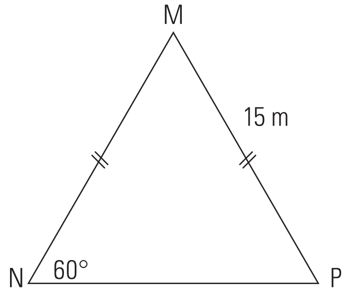
b) ΔABC is an isosceles triangle, because it has two sides of equal length. In an isosceles triangle, the interior angles opposite the congruent sides are equal, so $\angle$B equals $\angle$C. The sum of interior angles is 180°; use this to calculate $\angle$A.

$\angle A = 180° - \angle B - \angle C$
$\angle A = 180° - 53° - 53°$
$\angle A = 74°$

All of the interior angles of the triangle are less than 90°, so ΔABC is an acute triangle.

BUILD YOUR SKILLS

4. Use the following diagram to answer the questions below.

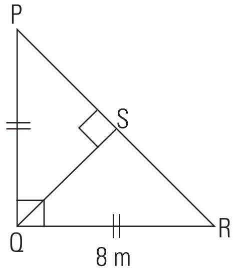


a) What is the measure of ∠M?

b) Classify ΔMNP by angle measure and by side length.

This style of house with an asymmetrical roof is called a saltbox. The portion of the house under the longer roof section is only one storey, while the front of the house under the shorter roof section is two storeys.

5. Use the following diagram to answer the questions below.

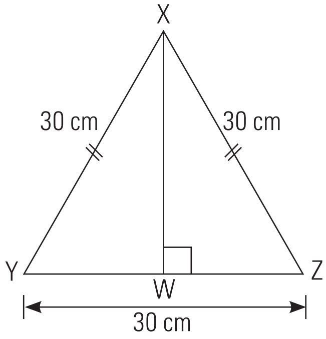


a) What is the length of PR?

b) Classify ΔPQR by angle measure and side length.

c) Classify ΔQRS by side length.

6. An equilateral triangle XYZ has a perimeter of 90 cm. What is its area?



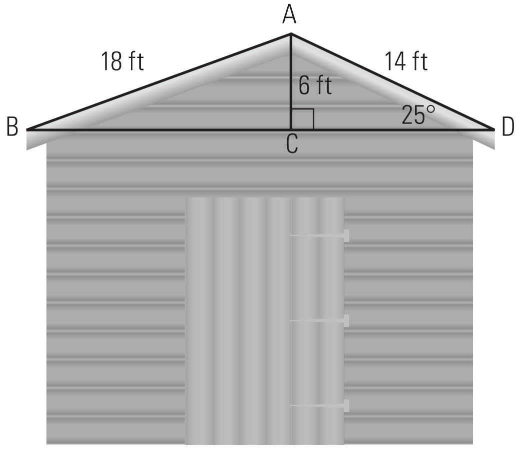
The area of a triangle is calculated using the following formula:

$$A = \frac{bh}{2}$$

where *b* is the base and *h* is the height.

Example 4

An asymmetrical roof on a shed has sections that are 18 feet and 14 feet long. The roof has a rise of 6 feet.



a) How wide is the building?

b) Classify ΔABC, ΔACD, and ΔABD by angle measure and side length.

Sheds can serve multiple purposes, such as a seasonal playhouse. Some interior designers have converted sheds into home offices, sitting rooms, and even guest suites.

SOLUTION

a) Use the Pythagorean theorem to find the length BC.

$$x^2 + y^2 = z^2$$
$$(\text{BC})^2 + (6)^2 = (18)^2$$
$$(\text{BC})^2 = (18)^2 - (6)^2$$
$$(\text{BC})^2 = 324 - 36$$
$$\text{BC} = \sqrt{324 - 36}$$
$$\text{BC} = \sqrt{288}$$
$$\text{BC} \approx 17.0 \text{ ft}$$

Side BC is about 17.0 feet long.

Use the Pythagorean theorem to solve for CD.

$$x^2 + y^2 = z^2$$
$$(\text{CD})^2 + (6)^2 = (14)^2$$
$$(\text{CD})^2 = (14)^2 - (6)^2$$
$$(\text{CD})^2 = 196 - 36$$
$$\text{CD} = \sqrt{196 - 36}$$
$$\text{CD} = \sqrt{160}$$
$$\text{CD} \approx 12.6 \text{ ft}$$

Side CD is about 12.6 ft long.

Calculate the width of the shed.

BD = BC + CD
BD = 17.0 + 12.6
BD = 29.6 ft

The shed is about 29.6 ft wide.

b) ΔABC and ΔACD are both right scalene triangles.

To classify ΔABD, calculate the measure of ∠DAB. ∠DAB is the sum of ∠DAC and ∠CAB.

∠DAC = 180° – 90° – 25°
∠DAC = 65°

The measure of ∠CAB can be calculated using the cosine ratio.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos\theta = \frac{6}{18}$$

$$\theta = \cos^{-1}\left(\frac{6}{18}\right)$$

$$\theta \approx 71°$$

Calculate the measure of ∠DAB.

∠DAB = ∠DAC + ∠CAB

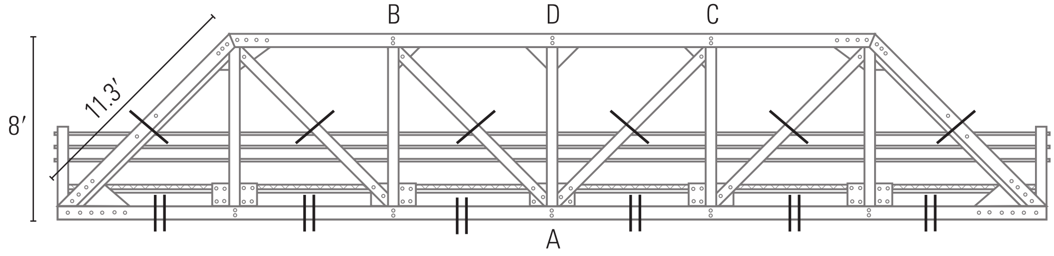
∠DAB = 65° + 71°

∠DAB = 136°

∠DAB is obtuse, so ΔABD is an obtuse scalene triangle.

BUILD YOUR SKILLS

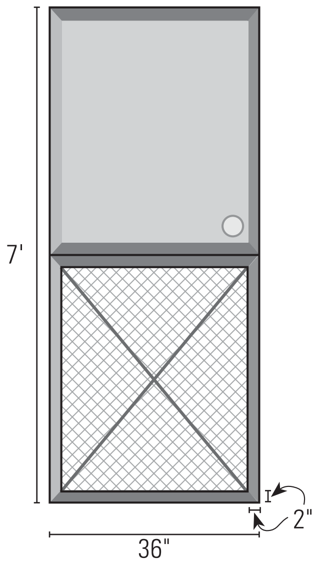
7. Arnold has designed a Pratt truss bridge, as shown in the diagram below.



a) How long is the bridge?

b) Calculate the interior angles of ΔABC.

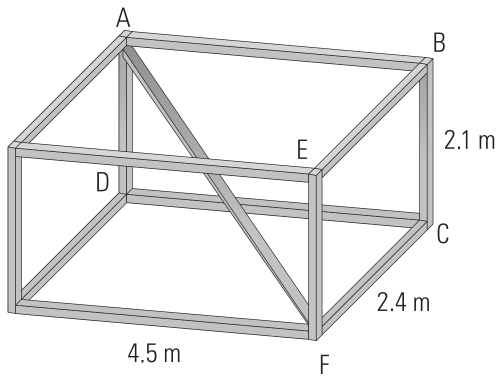
c) Classify ΔABC and ΔABD by angle measure and side length.



8. Annie is building a door, and she will be installing a square screen in the bottom half of the door. She plans to add diagonal cross-bracing across the area of the screen. The door is 36 inches wide and the screen has a 2-inch border around it. The total height of the door is 7 feet. What length of cross-bracing will she need?

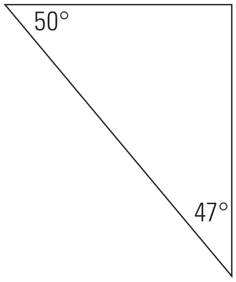
9. A large rectangular crane truss is 4.5 m long, 2.4 m wide, and 2.1 m tall. The truss has a brace that runs diagonally through it to provide additional support.

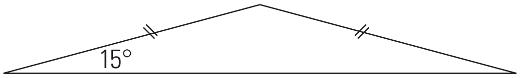
How long is the brace (AF)?

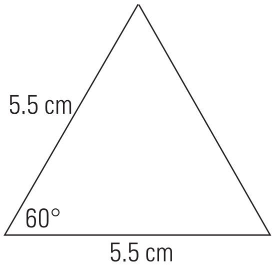


PRACTISE YOUR NEW SKILLS

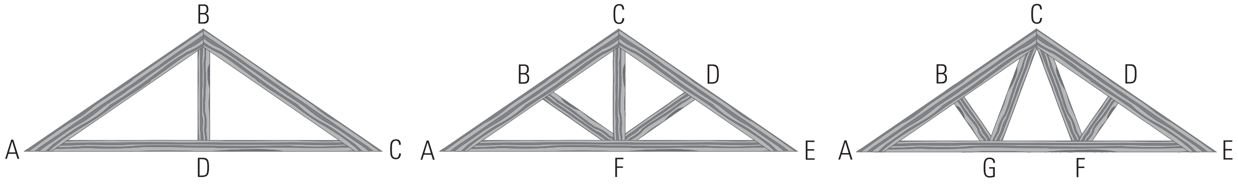
1. Classify each triangle by angle measure and side length. Give reasons for your classifications.

a) 

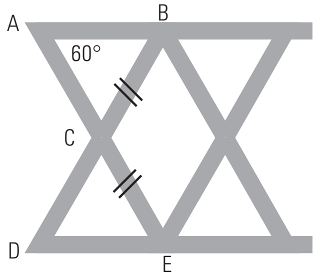
b) 

c) 

2. Roof trusses are triangular structures used to provide strength and stability to a building. The following roof trusses contain several triangles. Identify and classify all the triangles in each truss.



3. Nadine is building a fence with a lattice pattern. In the following diagram, AE and DB are both 30 cm long, and ∠BAC is 60°.



a) What is the measure of ∠CDE?

b) What is the length of DE?

c) If Nadine wants the fence to be about 2 m high, how many sections like this will need to be built vertically?

Quadrilaterals 5.2

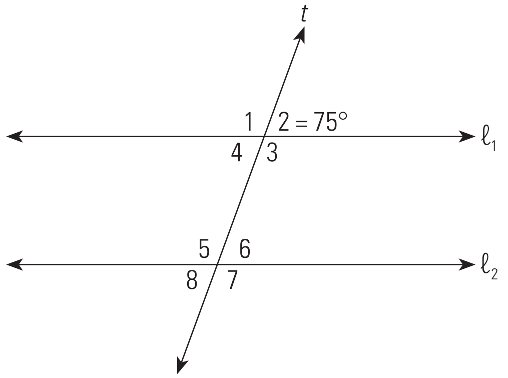
REVIEW: WORKING WITH PARALLEL LINES AND TRANSVERSALS

When parallel lines are intersected by a **transversal**, several angles are formed. When you know the lines are parallel and you know one of the angles, you can determine the rest of the angle measures because they are related to each other.

transversal: a line that intersects two or more lines at two distinct points

Example 1

In the following diagram, ℓ_1 is parallel to ℓ_2. What are the measures of the indicated angles?



SOLUTION

$\angle 1$ lies on the same line as $\angle 2$, so they are **supplementary**.

supplementary angles: two angles that add up to 180°

$\angle 1 = 180° - \angle 2$
$\angle 1 = 180° - 75°$
$\angle 1 = 105°$

$\angle 3$ is also on the same line as $\angle 2$, so they are supplementary. As well, $\angle 3$ is vertically opposite $\angle 1$, so $\angle 3$ and $\angle 1$ are equal.

$\angle 3 = 180° - \angle 2$
$\angle 3 = 105°$

$\angle 4$ is vertically opposite $\angle 2$, so they are equal.

$\angle 4 = \angle 2$
$\angle 4 = 75°$

$\angle 5$ and $\angle 1$ are corresponding angles. That is, they occupy the same position relative to the parallel lines and transversal at two different intersections. Corresponding angles are equal. Also, $\angle 5$ and $\angle 3$ are alternate interior angles—that is, angles formed on opposite sides of the transversal and parallel lines. Alternate interior angles are equal.

$\angle 5 = \angle 1$ and $\angle 5 = \angle 3$
$\angle 5 = 105°$

$\angle 6$ is an interior angle on the same side of the transversal as $\angle 3$. Interior angles on the same side of the transversal are supplementary. Also, $\angle 6$ is supplementary to $\angle 5$ because they form a straight line.

$\angle 6 = 180° - \angle 3$
$\angle 6 = 180° - 105°$
$\angle 6 = 75°$

$\angle 7$ corresponds to $\angle 3$, so they are equal.

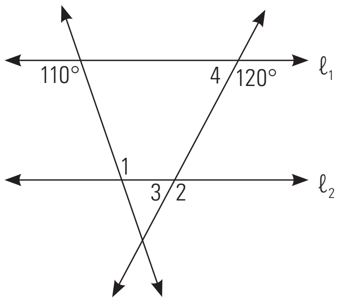
$\angle 7 = \angle 3$
$\angle 7 = 105°$

$\angle 8$ corresponds to $\angle 4$, so they are equal.

$\angle 8 = \angle 4$
$\angle 8 = 75°$

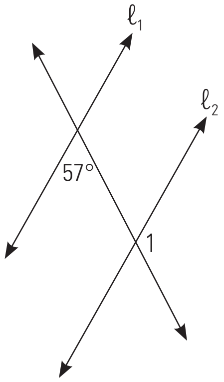
BUILD YOUR SKILLS

1. In the following diagram, ℓ_1 is parallel to ℓ_2. What are the measures of the indicated angles?



A protractor and a ruler can help you determine if lines are parallel.

2. What must be the size of ∠1 if ℓ_1 is parallel to ℓ_2?



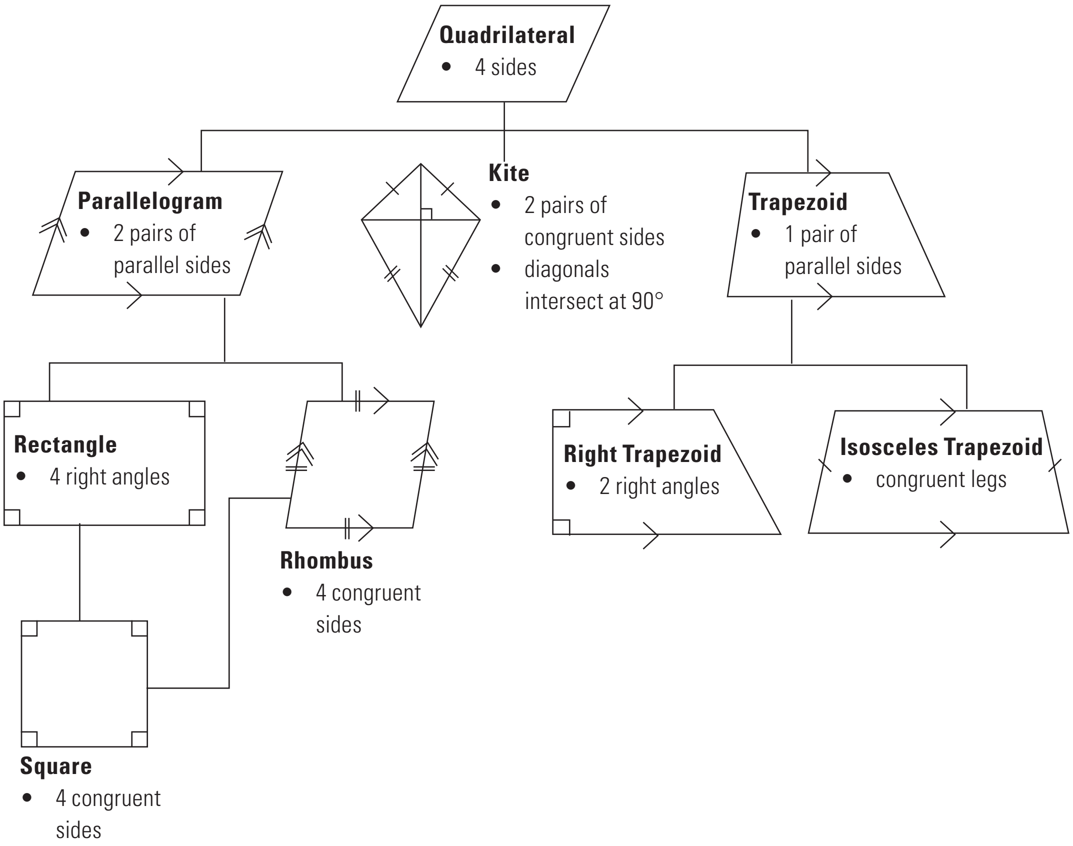
NEW SKILLS: WORKING WITH QUADRILATERALS

quadrilateral: a four-sided polygon lying on a single plane

A **quadrilateral** is a closed, 4-sided figure that lies on a single plane. That is, the sides meet at 4 vertices, and the figure lies flat. The sum of interior angles is 360°.

There are several kinds of quadrilaterals, each with additional properties, as shown in the following diagram. When you add another property, you get a new quadrilateral, and each quadrilateral has all the properties listed above it.

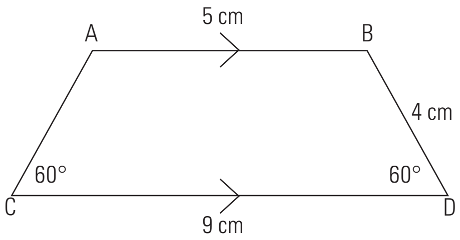
For more details, see page 191 of *MathWorks 12*.



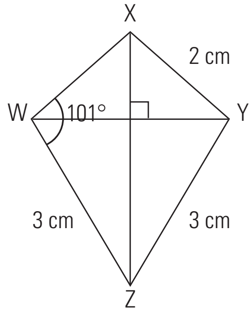
When classifying a quadrilateral, be as specific as possible. For example, although a square can be classified as a parallelogram, you should identify it as a square.

Example 2

a) Find AC, ∠A, and ∠B. What type of quadrilateral is ABCD?



b) Find WX and ∠XYZ. What is the sum of ∠WXY and ∠YZW?



SOLUTION

a) Quadrilateral ABCD is a trapezoid because it has a pair of parallel sides. The interior angles between the equal sides and one of the parallel sides are equal (∠C and ∠D), so it is an isosceles trapezoid. This means that AC is equal to BD. AC is 4 cm.

In a trapezoid, the interior angles on one of the legs are supplementary—that is, ∠A and ∠C add up to 180°, and ∠B and ∠D add up to 180°—because they are interior angles on the same side of the transversal. Calculate the measure of ∠A.

$\angle A = 180° - 60°$
$\angle A = 120°$

$\angle B = \angle A$
$\angle B = 120°$

b) Quadrilateral WXYZ is a kite, because the diagonals intersect at 90°, and the two longer sides (WZ and YZ) are of equal length. This means that the shorter sides are also of equal length.

WX = XY
WX = 2 cm

In a kite, the angles formed between the short and long sides (∠XWZ and ∠XYZ) are equal.

∠XYZ = ∠XWZ
∠XYZ = 101°

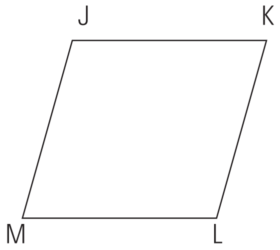
The sum of interior angles of a quadrilateral is 360°.

∠WXY + ∠YZW = 360° – ∠XYZ – ∠XWZ
∠WXY + ∠YZW = 360° – 101° – 101°
∠WXY + ∠YZW = 158°

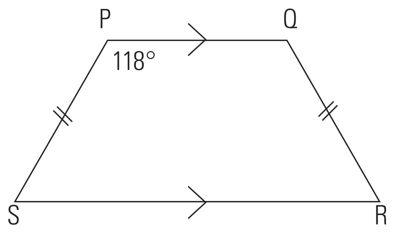
BUILD YOUR SKILLS

The first complex protractor, called the three-arm protractor, was created for plotting the position of a boat on navigational charts. It was invented in 1801 by US naval captain Joseph Huddart.

3. Using a ruler and a protractor, measure the side lengths and angles of the following quadrilateral. What kind of quadrilateral is it?

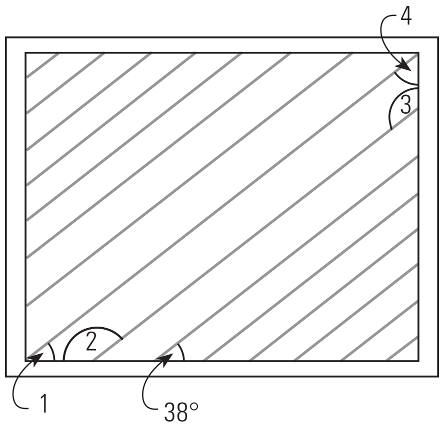


4. Using your knowledge of the properties of quadrilaterals, find the measures of the missing angles. What kind of quadrilateral is this?



Example 3

A rectangular window is to be constructed with strips of coloured glass that run parallel to each other and diagonally as shown. At what angles must the glass cutter cut each of the indicated angles?



This stained glass window uses many parallel lines and geometric figures to create an attractive design.

SOLUTION

The strips of glass are all parallel, so $\angle 1$ is corresponding to the 38° angle.

$\angle 1 = 38°$

$\angle 2$ is supplementary to an angle of 38°.

$\angle 2 = 180° - 38°$
$\angle 2 = 142°$

The strips of glass are isosceles trapezoids. This means that $\angle 1$ and $\angle 4$ are equal, and $\angle 2$ and $\angle 3$ are equal.

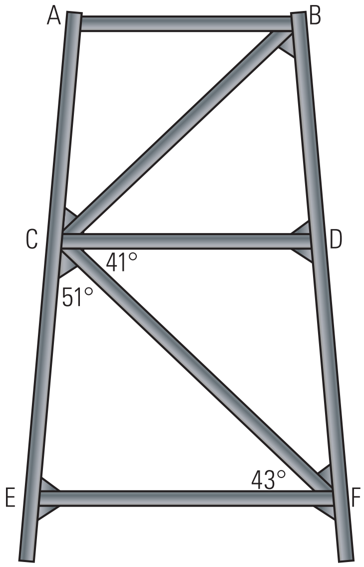
$\angle 3 = \angle 2$
$\angle 3 = 142°$

$\angle 4 = \angle 1$
$\angle 4 = 38°$

The glass cutter must cut $\angle 1$ and $\angle 4$ at 38°, and $\angle 2$ and $\angle 3$ at 142°.

BUILD YOUR SKILLS

5. A transmission tower is being built to provide power to a new residential subdivision. On the tower, braces AB, CD, and EF must all be parallel to the ground.



Transmission towers use a lattice framework to create a strong, stable structure.

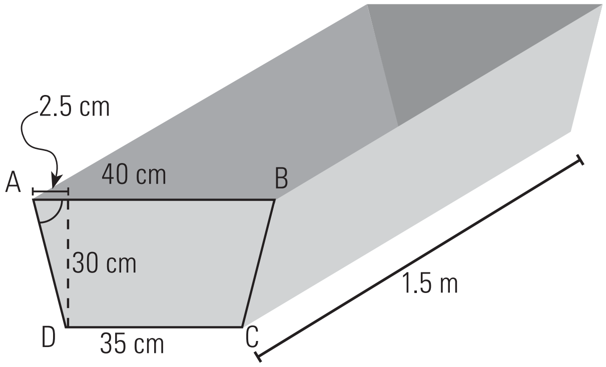
a) What adjustment needs to be made to ∠DCF for CD to be parallel to the ground? (The adjustments will be made by raising or lowering CD at point D.)

b) What will be the measure of ∠CAB? Why?

c) What type of quadrilateral is ABFE?

Window planter boxes let apartment dwellers enjoy gardening without a yard.

6. Leslie is building a planter for her front porch. The planter is a trapezoidal prism.



a) At what angle does she need to cut ∠A?

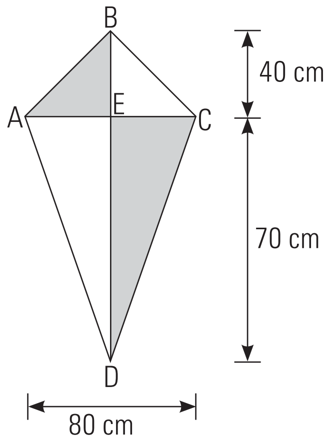
b) How much soil (in cm³) can the planter hold?

HINT:

The area of a trapezoid can be calculated as follows, where *a* and *b* are the parallel sides and *h* is the height:

$A = \frac{1}{2}(a + b)h$

7. Ian is sewing a kite that he plans to take to the annual Windscape Kite Festival in Swift Current, SK.



a) What are the measures of ∠BCE and ∠CBE? What kind of triangle is BEC?

b) What are the measures of ∠EAD and ∠ECD?

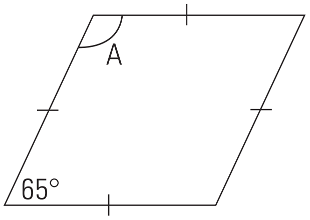
c) What are the measures of ∠BCD and ∠BAD?

d) How much of each colour of material (grey and white) is needed to make the kite (in cm^2)?

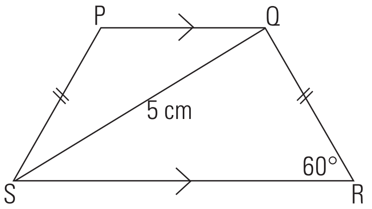
PRACTISE YOUR NEW SKILLS

1. Solve for the indicated length or angle, and identify the type of quadrilateral.

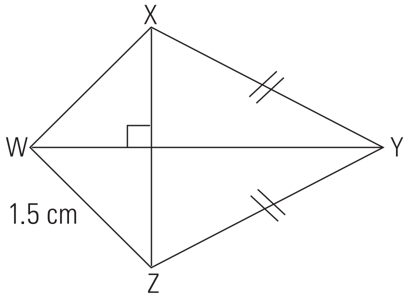
a) ∠A



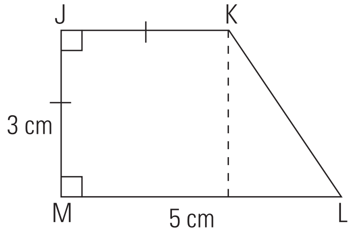
b) PR



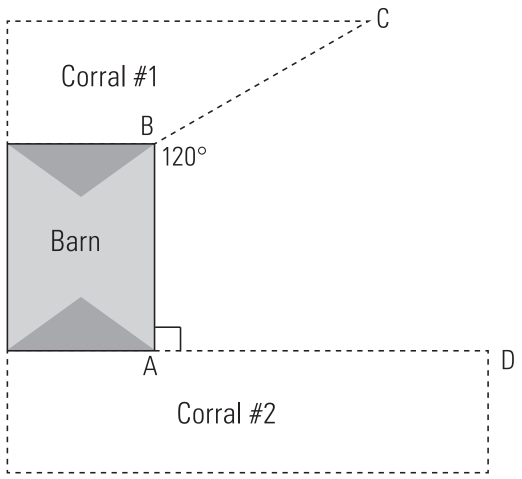
c) WX



d) ∠L



2. A farmer plans to build an enclosure behind a barn, between existing corrals. He wants to add a fence from point C that runs parallel to the barn (wall AB) towards fence AD.

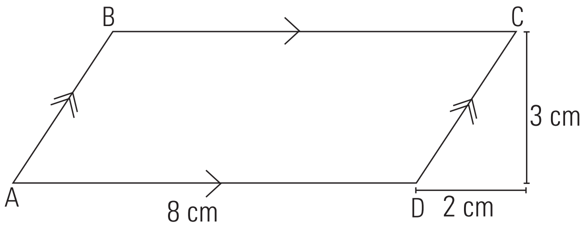


a) How can he ensure that the new fence is parallel to AB?

b) What type of quadrilateral will be formed by the enclosure when the new fence is built?

Horseback riding is often used as a therapeutic activity for children and adults with disabilities. Horseback riding can help improve balance, increase strength, and develop communication skills and self-esteem.

3. a) What is the perimeter of parallelogram ABCD?



b) What is the measure of ∠ADC?

c) What is the measure of ∠DCB?

Regular Polygons 5.3

NEW SKILLS: WORKING WITH REGULAR POLYGONS

A **polygon** is a closed figure formed by joining three or more line segments at their endpoints. All the line segments must lie on a plane. They are classified according to the number of sides. Some common polygons are listed in the following table.

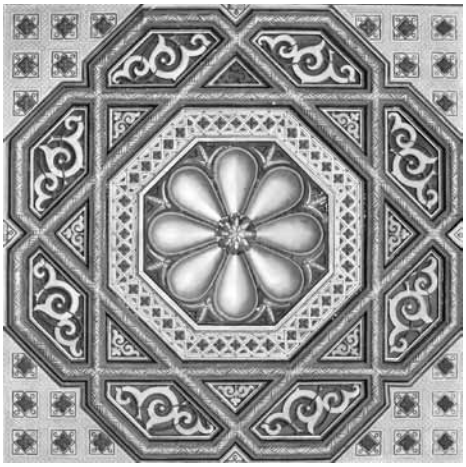
polygon: a closed figure of three or more sides lying on a single plane

COMMON POLYGONS	
Name	*Number of sides*
triangle	3
quadrilateral	4
pentagon	5
hexagon	6
heptagon	7
octagon	8
nonagon	9
decagon	10
dodecagon	12

If all sides of a polygon are equal and all angles are equal, it is called a **regular polygon**.

regular polygon: a polygon with congruent side lengths and congruent interior angles

For more details, see page 202 of *MathWorks 12*.



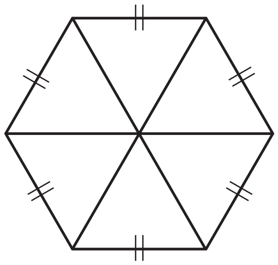
What types of polygons can you identify in this tile pattern?

Example 1

What are the sizes of the interior angles of a regular hexagon?

SOLUTION

A regular hexagon is a 6-sided figure with all sides of equal length. Join pairs of opposite vertices. They will meet at the centre of the hexagon, forming 6 triangles.



The sum of the angles of a triangle is 180°. Calculate the sum of the angles in the 6 triangles.

$6 \times 180° = 1080°$

This includes the angles at the centre, which form a full turn of 360°, so subtract this from the total.

$1080° - 360° = 720°$

This is the sum of interior angles in a hexagon. Since all 6 interior angles are equal, divide by 6.

$720° \div 6 = 120°$

Each of the interior angles of a regular hexagon is 120°.

ALTERNATIVE SOLUTION

You can use the following formula for finding the sum of interior angles of a polygon, where n is the number of sides, then divide by the number of interior angles as above.

sum of interior angles $= 180°(n - 2)$

BUILD YOUR SKILLS

1. Melanie says that since a dodecagon has twice as many angles as a hexagon, the angles in a regular dodecagon will be half the size of those in a regular hexagon. Is she right or wrong? Calculate the size of each of the angles of a regular dodecagon to verify your conclusion.

2. Fill in the table below, showing the size of each of the interior angles for each regular polygon.

INTERIOR ANGLE SIZE IN REGULAR POLYGONS

Regular polygon	*Angle size*
Triangle	
Square	
Pentagon	
Hexagon	
Heptagon	
Octagon	
Nonagon	
Decagon	
Dodecagon	

3. What regular polygon is approximated by joining the ends of the petals of the flower? What is the angle at each vertex?

Example 2

The Giant's Causeway is an area in Ireland that is made up of about 40 000 columns of cooled lava that are mostly hexagonal columns.

If the columns forms regular hexagons, how many would join together to form a complete tiling, leaving no gaps between columns?

SOLUTION

In the previous example, you calculated that the interior angles of a regular hexagon are 120°. Calculate how many of these angles would fit around a common point.

There are 360° around a common point. Divide by 120°.

$360° \div 120° = 3$

Three regular hexagons will fit perfectly around a central point, so three columns would have to join together to form a complete tiling, with no gaps.

BUILD YOUR SKILLS

4. Example 2 shows that regular hexagons fit perfectly around a common point; this means that it would be possible to tile a floor with tiles in the shape of regular hexagons. What other regular polygons can be used to tile a floor? Give two possibilities. Explain your answer.

.

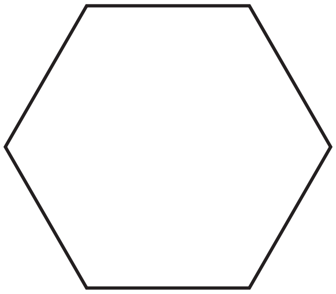
5. Octagonal houses were popular in the 1850s, and a number of historical octagonal houses exist in Canada and the United States. One such building is the Captain George Scott House in Wiscasset, Maine, USA.

a) If the house is a regular octagon, what is the sum of angles at the vertices where the exterior walls meet?

b) If the front wall of the house is 10 feet wide, what is the perimeter of the house?

6. The floor of a gazebo has the shape of a regular hexagon.

   a) The floor is painted in a pattern that divides the hexagon into triangles. Draw the lines needed to divide the floor into 6 congruent triangles.



   b) What are the interior angles of the triangles? Classify the triangles by side length.

   c) If the sides of the regular hexagon each measure 3 metres, what is the area of the floor?

PRACTISE YOUR NEW SKILLS

1. Each angle of a regular polygon has a measure of 162°. How many sides does the polygon have?

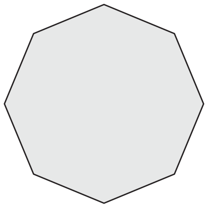
2. Alvin used a scale of 1:48 to build a model of a regular pentagonal building. The sides of the building are 24 feet long.

   a) What is the size of the building's interior angles?

   b) What is the length of one side of the model?

   c) What is the size of the model's interior angles?

3. Prince Edward Island is known for its many lighthouses. The Seacow Head Lighthouse was built in 1864. Its base is in the shape of a regular octagon. Each side of the regular octagon is 3.4 m long. What is the area of the floor? (Hint: Divide the octagon into triangles and use the tangent ratio to calculate the triangle height.)



Seacow Head Lighthouse, near Bedeque, PEI, was named for the walrus, or sea cow. It was built in 1894, but had to be relocated in 1979 due to shore erosion.

4. The Barbados dollar is in the shape of a regular polygon, as shown in the photograph.

The Barbados dollar.

   a) What type of polygon is it, and what is the sum of the interior angles?

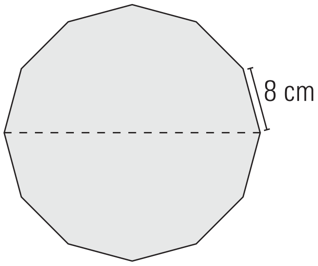
   b) What is the size of each interior angle?

5. Shelby is building her mother a special wall clock. She wants to make it in the shape of a regular dodecagon, like the one shown here.

a) What is the measure of each interior angle of a dodecagon?

Can you figure out what time this clock is showing?

b) Each side of the dodecagon is 8 cm. Divide the dodecagon into triangles and calculate the base angles of the triangles. Use this information to calculate the width of the clock face, from the 9 o'clock position to the 3 o'clock position.



CHAPTER TEST

1. For each of the properties given in the table below, list which of the following polygons have the given property.

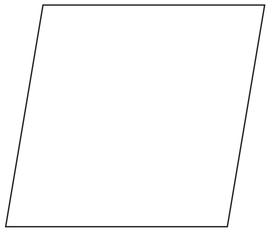
   Polygons: trapezoid, parallelogram, rhombus, isosceles trapezoid, rectangle, square, equilateral triangle, isosceles triangle, regular pentagon

PROPERTIES OF POLYGONS

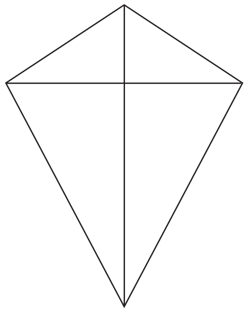
At least two equal sides	*At least two congruent angles*	*At least one pair of parallel sides*	*Interior angles add up to 360°*	*A regular polygon*

2. Measure and label the side lengths and angles of the figure. Name the shape.

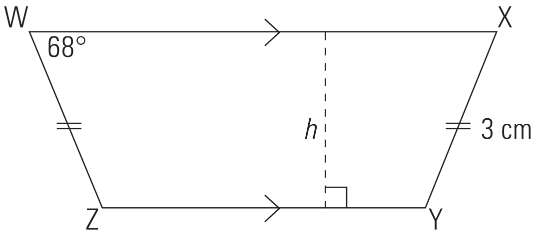
   a)



   b)



3. Using the diagram below, answer the following questions.



a) What is the measure of ∠Y?

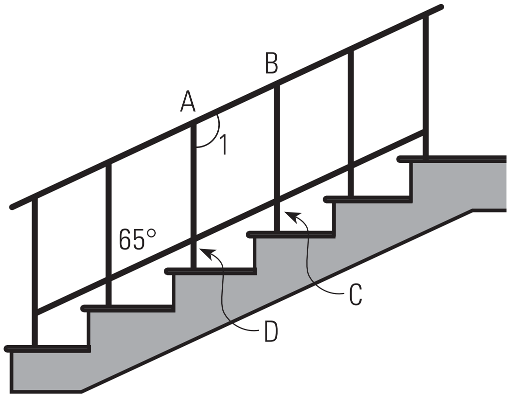
b) What is the length of *h*?

c) What type of quadrilateral is WXYZ?

A dormer is a structural feature that protrudes from a sloping roof. The term dormer comes from the Latin word for sleep—an indication of the popularity of dormer windows in bedrooms.

4. The dormer window on a house is in the shape of an isosceles triangle. The angle at the top is 76°. What are the measures of the angles at the base?

5. Examine the diagram of a stairwell railing below.

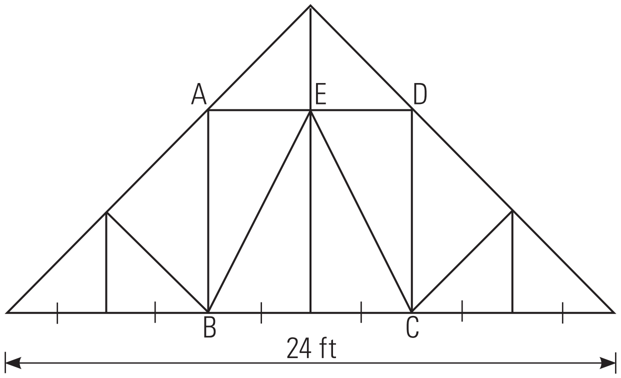


a) What is the measure of ∠1?

b) What type of quadrilateral is ABCD?

6. Omar just bought a new hot tub for his back yard. It is in the shape of a regular hexagon. Each side of the hexagon is 2.5 feet long. What area in square feet will the hot tub cover?

7. A roof truss for a garage is designed as shown below. ABCD is a square.



a) Calculate the length of BE.

b) Classify ΔABE by side length and angle measure.

c) Classify ΔBEC by side length and angle measure.

Chapter 6

Transformations

The blades of a wind turbine are an example of a rotation—a type of transformation. The three blades are identical, but rotated about a central point. This wind turbine is located at the top of Grouse Mountain in North Vancouver, BC. It generates power for use by the ski resort. There is an observation platform at the top of the tower that allows visitors a view of downtown Vancouver below.

Single Transformations 6.1

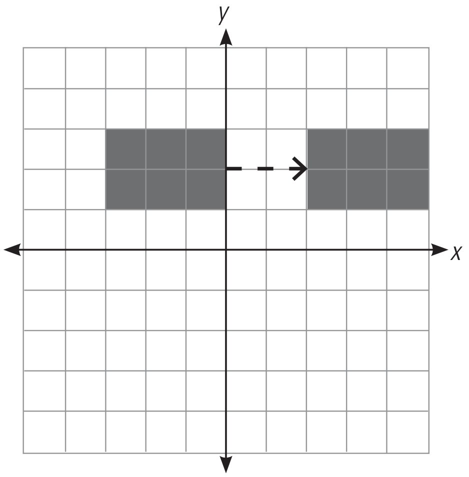
NEW SKILLS: WORKING WITH TRANSFORMATIONS

When you move a shape or object from one location to another—for example, by sliding it sideways or rotating it around point—you are performing a **transformation**. In industry and business, transformations are used in many situations. In this chapter, you will work with four types of transformations, as shown on the following pages.

transformation: the movement of a shape or an object by changing its position, size, and/or shape

translation: the horizontal and/or vertical movement of a shape or an object

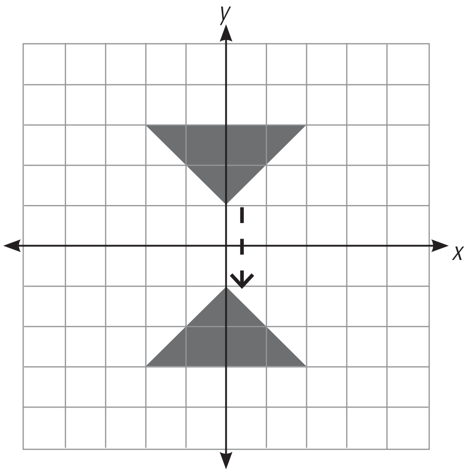
- **translation:** moving a shape or object horizontally and/or vertically; a translation does not change the size or shape of the object, just the location



reflection: a mirror image created by transforming an original image over a reflection line

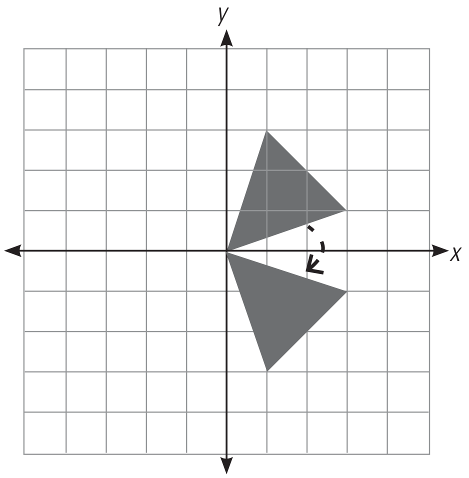
reflection line: a line across which a figure is reflected

- **reflection:** creating a mirror image of a shape or object over a **reflection line**



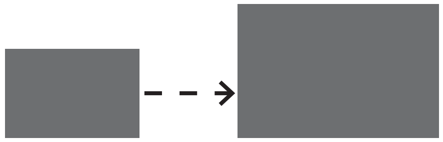
rotation: the movement of a shape or an object around a fixed point

- **rotation:** moving a shape or object around a fixed point



- **dilation:** enlarging or reducing a shape or object to create a similar figure

dilation: the enlargement or reduction of a shape or object



For more details, see pages 216–217 of *MathWorks 12*.

NEW SKILLS: WORKING WITH TRANSLATIONS

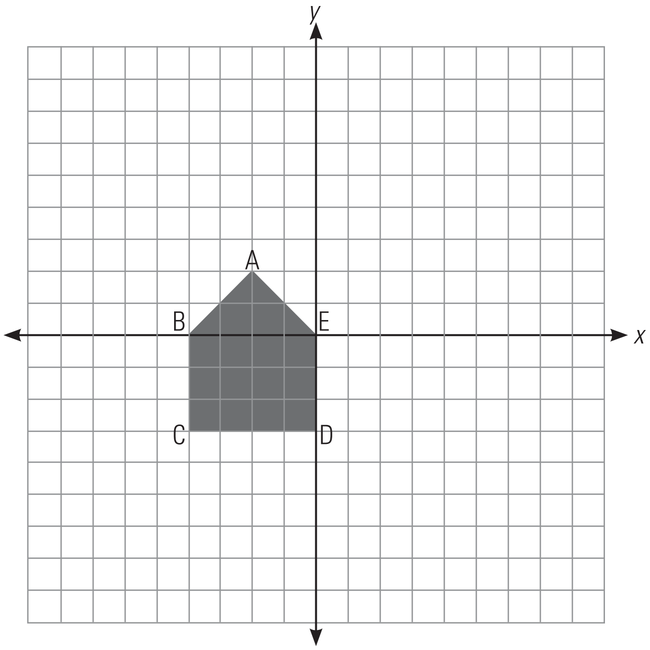
A translation occurs when a shape or image moves up, down, right, or left, without changing size or shape. Each vertex of the shape will move the same distance in the same direction.

For more details, see page 216 of *MathWorks 12*.

Example 1

Translate the following shape 6 units to the right and 2 units down. What are the coordinates of the resulting vertices?

You may wish to trace shape ABCD on a separate piece of paper and cut it out, then use the cut-out to help you visualize the translation. You can move your cut-out on the coordinate plane.



SOLUTION

The original shape has the following vertices: A(–2, 2), B(–4, 0), C(–4, –3), D(0, –3), and E(0, 0).

The shape is to be translated 6 units to the right and 2 units down. For vertex A(–2, 2), this means:

- It will move from –2 horizontally to a position 6 units to the right, or to 4.
- It will move from 2 vertically to a position 2 units down, or to 0.

A(–2, 2) moves to A′(4, 0). Every other vertex will move the same directions and distances.

B(–4, 0):	horizontally:	–4 + 6 = 2
	veritcally:	0 – 2 = –2

B(–4, 0) moves to B′(2, –2).

C(–4, –3):	horizontally:	–4 + 6 = 2
	vertically:	–3 – 2 = –5

C(–4, –3) moves to C′(2, –5).

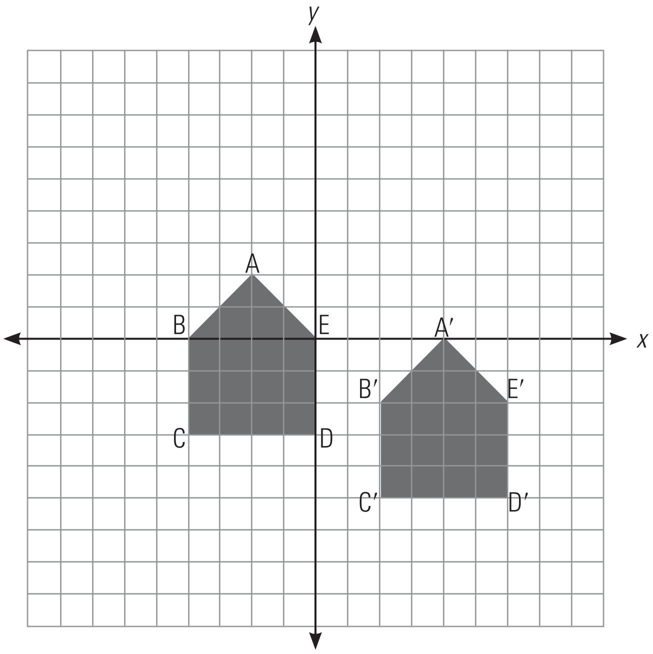
D(0, –3):	horizontally:	0 + 6 = 6
	vertically:	–3 – 2 = –5

D(0, –3) moves to D′(6, –5).

E(0, 0):	horizontally:	0 + 6 = 6
	vertically:	0 – 2 = –2

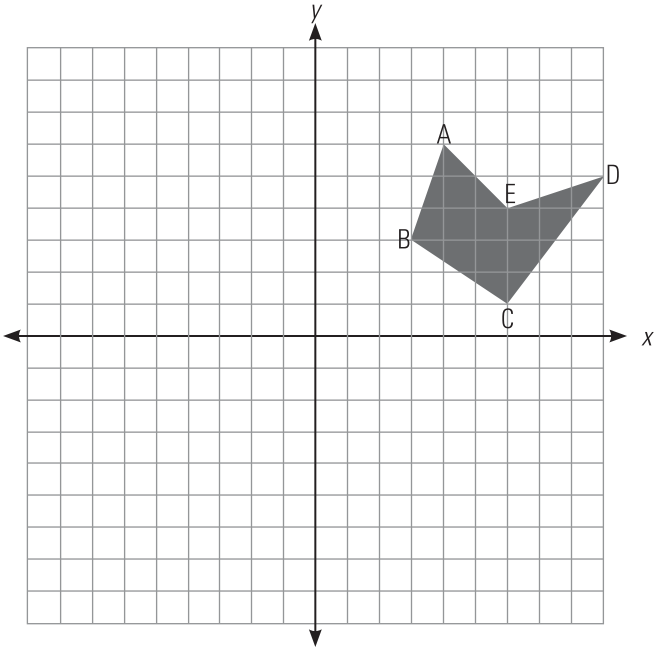
E(0, 0) moves to E′(6, –2).

A prime mark (′) is often used to indicate a transformed point.



BUILD YOUR SKILLS

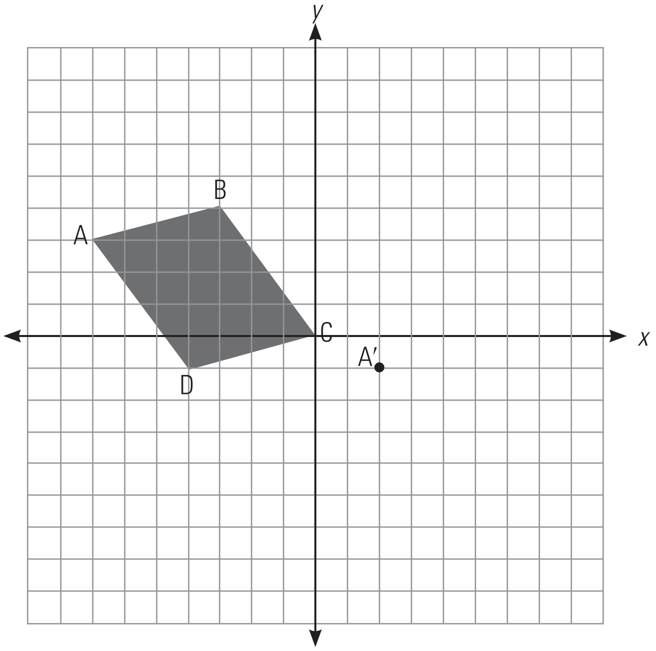
1. Translate the shape 5 units to the left and 3 units down. What are the coordinates of the vertices of the translated image?



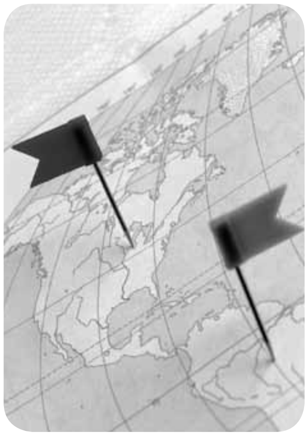
Moving a knight playing piece on a chess board is an example of translation. The knight can move up and left or right.

2. Parallelogram ABCD is translated such that vertex A moves to A′(2, –1).

   a) Draw the other vertices of the translated parallelogram and state their coordinates.

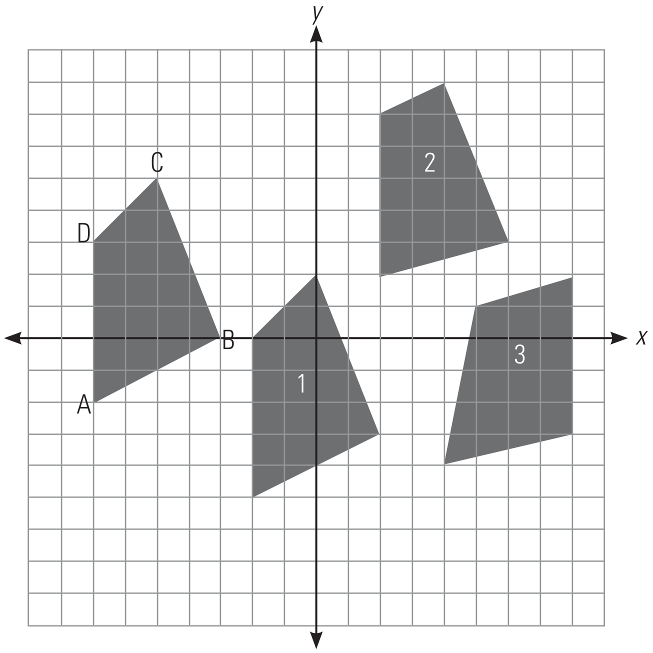


b) How far does the parallelogram move horizontally and vertically?

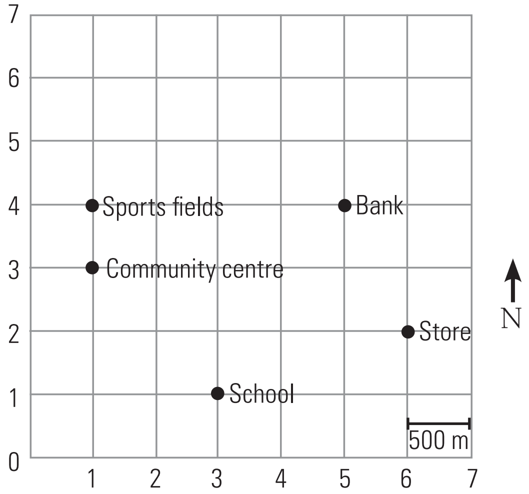


Maps often use grid lines to help you find locations and interpret the scale of the map.

3. Which shape—1, 2, or 3—is a translation of ABCD? How far did ABCD move horizontally and vertically?



4. Michèle is using the map below to find her way around town during a holiday.



a) Michèle is at the store and needs to walk to the bank. How many grid units, and in which direction, does she need to travel?

b) How many metres north and west would you have to walk to get from the school to the sports fields?

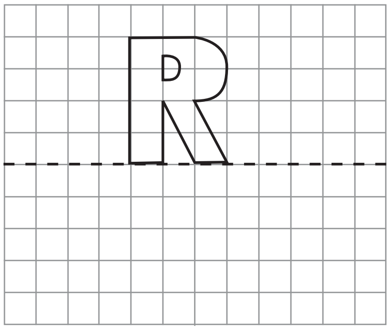
NEW SKILLS: WORKING WITH REFLECTIONS

When you look in a mirror, you see a reflection of yourself. A reflection is a transformation of a shape or an object that creates an identical image across a reflection line. The points of the image will be the same distance from the reflection line as the points of the original shape. If you were to draw a shape and its reflection on a piece of paper, and then if you folded the paper along the reflection line, the original shape and its image would match up.

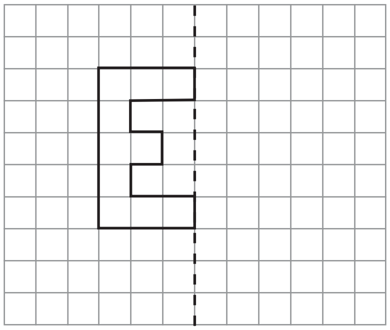
For more details, see page 220 of *MathWorks 12*.

Example 2

a) Draw a mirror image of this letter, reflected over a horizontal line.

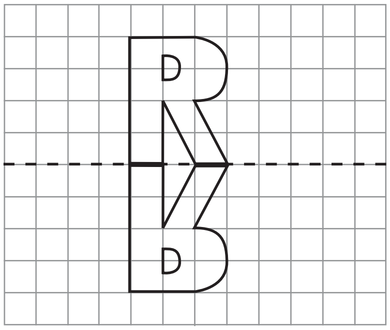


b) Draw a mirror image of this letter, reflected over a vertical line.

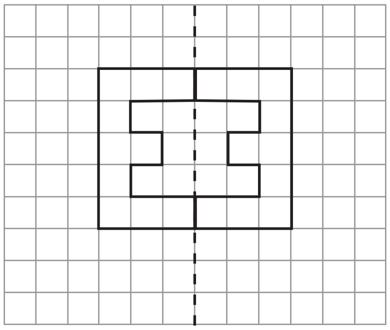


SOLUTION

a) When the shape is reflected over a horizontal line, the image will appear upside-down. Each point on the reflected image (including those not at the vertices) will be the same distance from the reflection line as in the original shape, but on the opposite side of the line.



b) When the shape is reflected over a vertical line, the image will appear backwards. Here, too, each point on the reflected image will be the same distance from the reflection line as in the original shape.



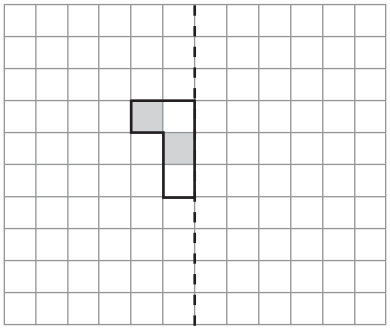
BUILD YOUR SKILLS

5. Write your first name in capital letters, then reflect it over a horizontal reflection line.

Where is the reflection line in this photo?

6. Are there any letters of the alphabet, when written as capital letters, that look the same after they have been reflected over a horizontal line (that is, when they are flipped upside down)? If so, what are they?

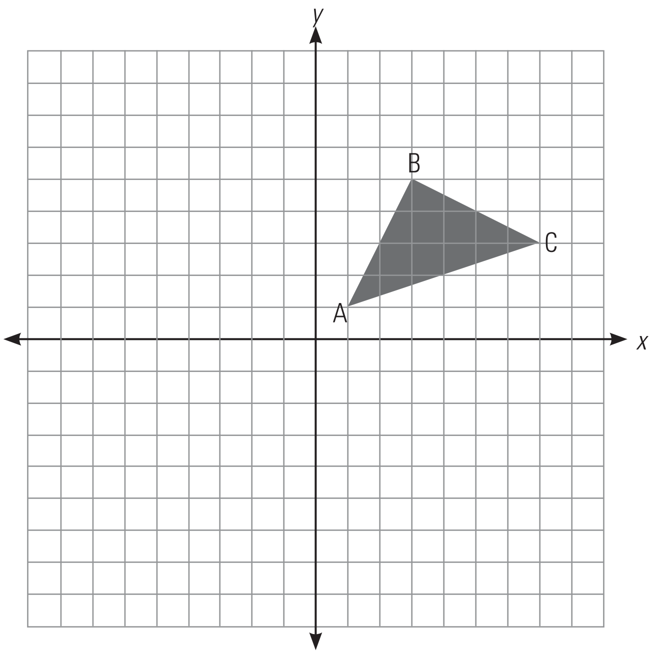
7. Reflect the following image over the indicated vertical reflection line.



Example 3

a) Reflect triangle ABC over the *y*-axis. Label the vertices A′, B′, and C′.

b) Reflect triangle ABC over the *x*-axis. Label the vertices A″, B″, and C″.



SOLUTION

a) When a shape is reflected over the *y*-axis, the *x*-coordinates will change signs (from positive to negative, or negative to positive), and the *y*-coordinates will remain the same.

A(1, 1) will become A′(–1, 1).

B(3, 5) will become B′(–3, 5).

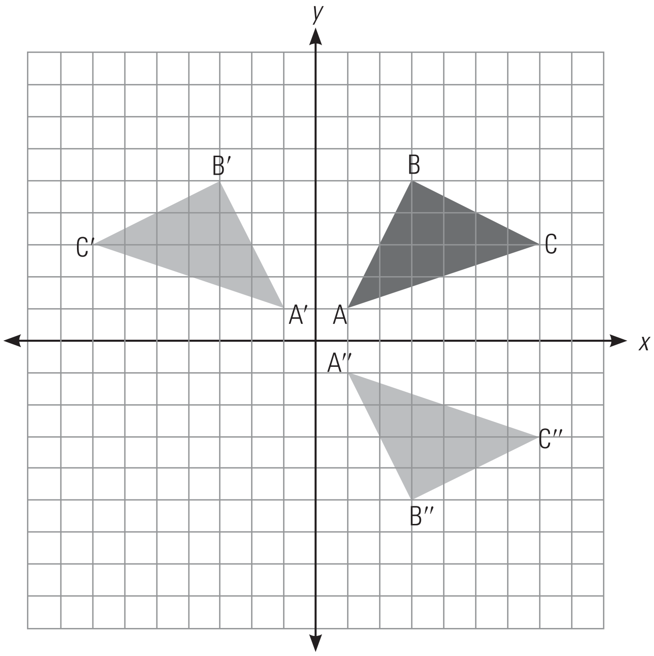
C(7, 3) will become C′(–7, 3).

b) When a shape is reflected over the *x*-axis, the *x*-coordinates will remain the same, and the *y*-coordinates will change signs.

A(1, 1) will become A″(1, –1).

B(3, 5) will become B″(3, –5).

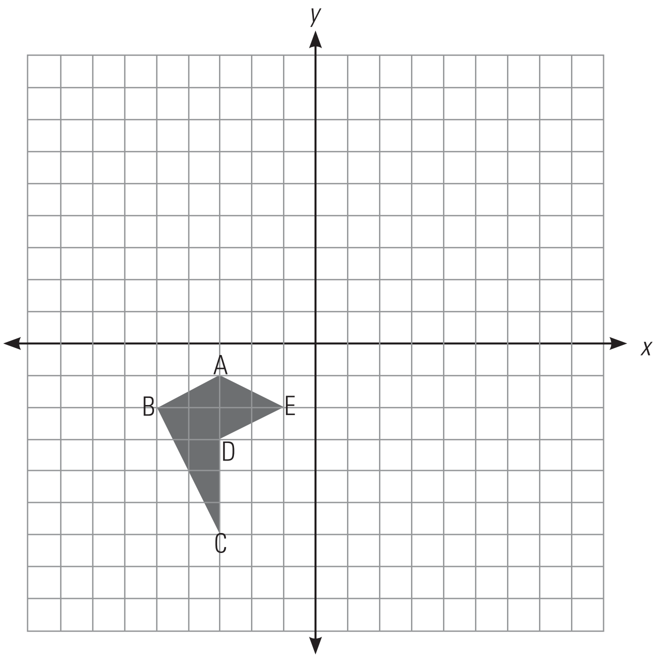
C(7, 3) will become C″(7, –3).



BUILD YOUR SKILLS

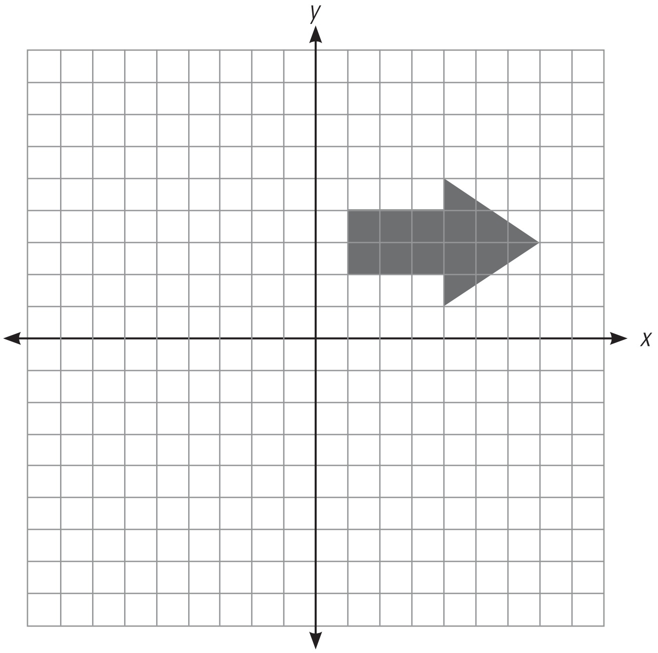
8. a) Reflect shape ABCDE over the *y*-axis and label its vertices A′, B′, C′, D′, and E′.

   b) Reflect shape ABCDE over the *x*-axis and label its vertices A″, B″, C″, D″, and E″.



You can buy kits to allow you to create your own T-shirt designs. The kits come with heat-transfer paper; you just print your design on the special paper as a vertical reflection, then iron the image onto your T-shirt.

9. Rupinder is creating an image that will be printed on a T-shirt. In order for the image to appear correctly on the T-shirt, he needs to draw a mirror image of it, reflected over the *y*-axis. Draw the reflection.



Example 4

In the previous example, you reflected an image over a reflection line to create two symmetrical images. It is also possible for a single image to be symmetrical about a line—that is, the image can be folded in half to create mirror images on either side of the line.

Examine the following letters of the alphabet and determine which are symmetrical about a line. Draw the line of symmetry if it exists.

A C R D H U

SOLUTION

Examine the letters to see which can be folded in half to create symmetrical sides.

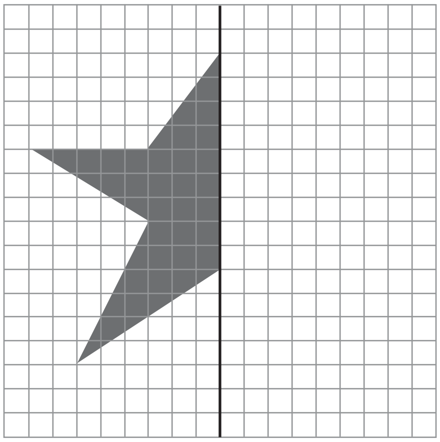
Letters A, H, and U have a vertical line of symmetry.

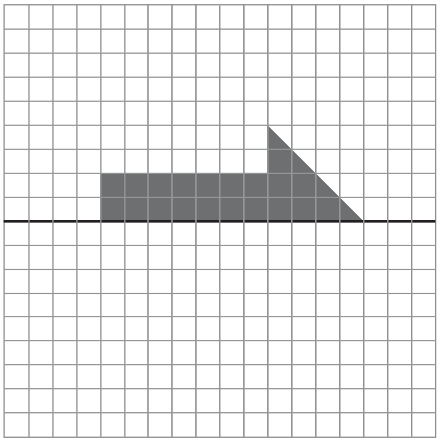
Letters C, D, and H have a horizontal line of symmetry.

Letter R has no lines of symmetry.

BUILD YOUR SKILLS

10. Complete each of the diagrams below, drawing the reflection across the line of symmetry.

a) 

b) 

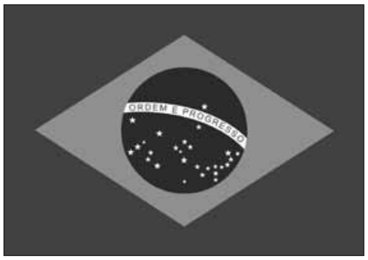
11. Examine the following national flags. Draw in any lines of symmetry that exist. Consider shape only, not colour.

Canada

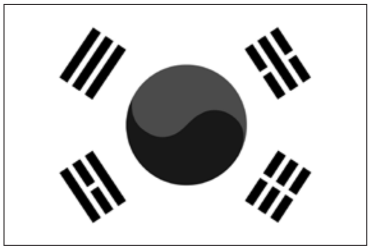
Austria

United Kingdom

India



Brazil


South Korea

12. In Example 2 you looked at images reflected over a line, and in Example 3 you looked at images that are symmetrical. How are the two ideas related?

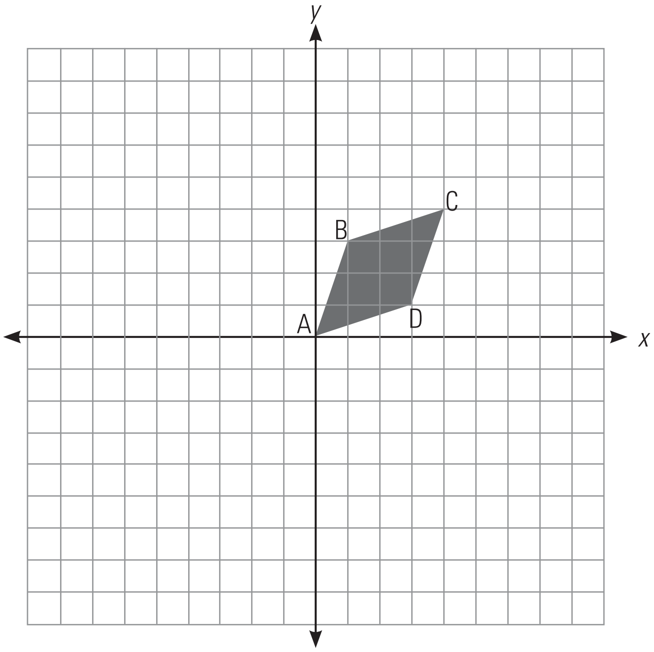
NEW SKILLS: WORKING WITH ROTATIONS

A rotation is a transformation in which points of an object are turned about a fixed point called the centre of rotation. In order to perform the rotation, you must know the degree and direction of the rotation.

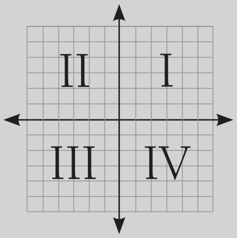
For more details, see page 222 of *MathWorks 12*.

Example 5

Using the origin as the centre of rotation, rotate the shape 90°, 180°, and 270° clockwise. State the coordinates of the translated vertices.



The four quadrants on a coordinate plane are numbered as follows:



SOLUTION

Start by identifying the coordinates of the vertices of the original shape: A(0, 0), B(1, 3), C(4, 4), D(3, 1).

90° rotation: When the shape rotates 90° clockwise about the origin, it will move from the first quadrant to the fourth quadrant, so the *x*-coordinates will be positive numbers and the *y*-coordinates will be negative numbers. The vertices will be the same distance from the origin as in the original shape.

Vertex A(0, 0) remains in the same location. Vertex B(1, 3) rotates to (3, –1). Vertex C(4, 4) rotates to (4, –4). Vertex D(3, 1) rotates to (1, –3). This shows that, when a point rotates 90° clockwise about the origin, the *x*-coordinate changes its sign, then the *x*-coordinate and the *y*-coordinate switch positions.

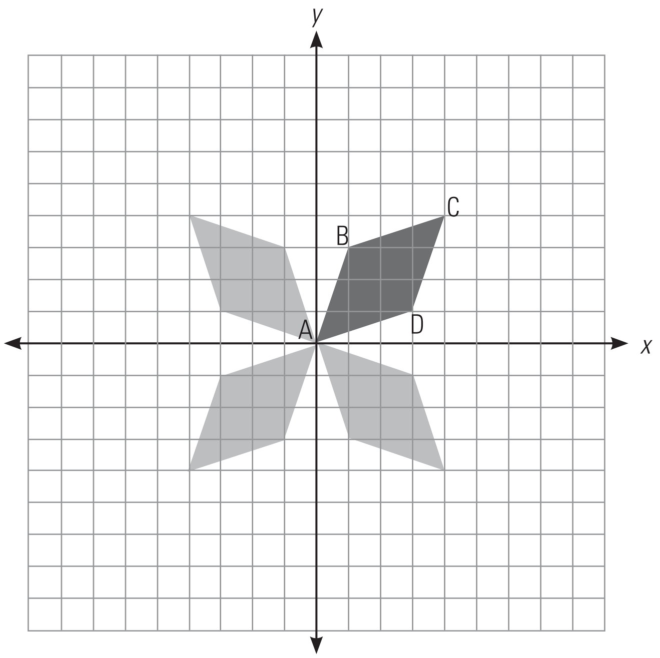
You may wish to trace shape ABCD on a separate piece of paper and cut it out, then use your cut-out to help you visualize the rotation. Hold the shape at point A, and rotate the cut-out around that fixed point.

180° rotation: When the shape rotates 180° clockwise about the origin, it will move from the first quadrant to the third quadrant. The *x*- and *y*-coordinates will both be negative numbers.

Vertex A(0, 0) remains in the same location. Vertex B(1, 3) rotates to (–1, –3), vertex C(4, 4) rotates to (–4, –4), and vertex D(3, 1) rotates to (–3, –1). That is, when a point rotates 180° about the origin, the coordinates stay in the same position but change signs.

270° rotation: When the shape rotates 270° clockwise about the origin, it will move from the first quadrant to the second quadrant. The *x*-coordinate will be a negative number and the *y*-coordinate will be positive.

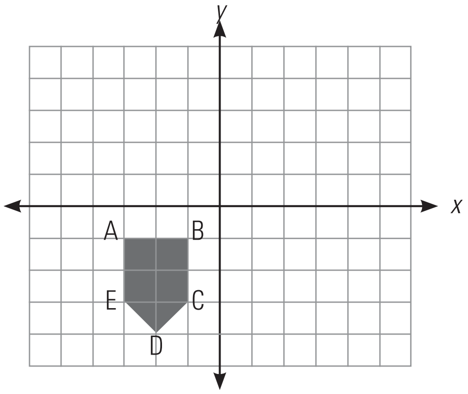
Vertex A(0, 0) remains in the same location. Vertex B(1, 3) rotates to (–3, 1), vertex C(4, 4) rotates to (–4, 4), and vertex D(3, 1) rotates to (–1, 3). That is, when a point rotates 180° about the origin, the *y*-coordinate changes its sign, then the *x*-coordinate and the *y*-coordinate switch positions.



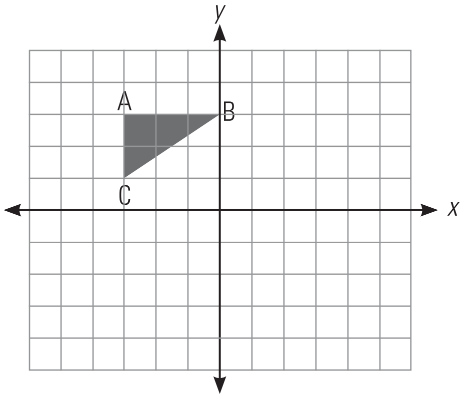
BUILD YOUR SKILLS

13. Rotate the following shapes about the origin in the direction and degrees indicated.

a) 180° clockwise

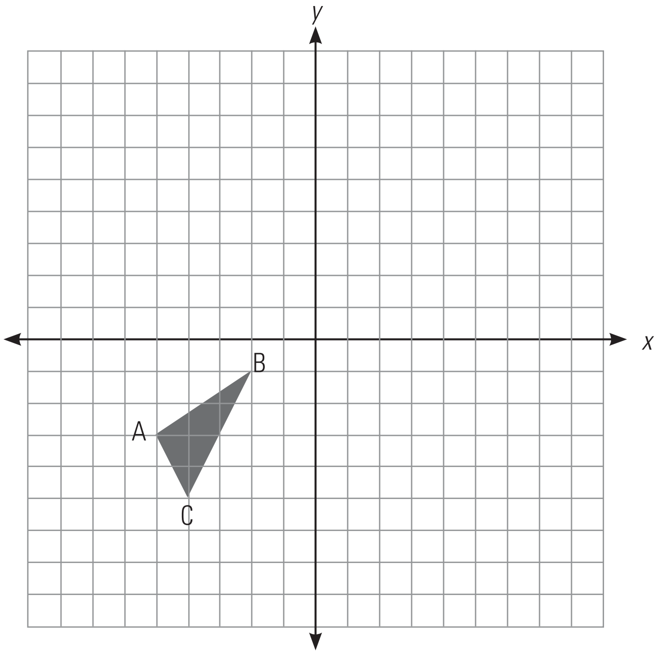


b) 90° clockwise



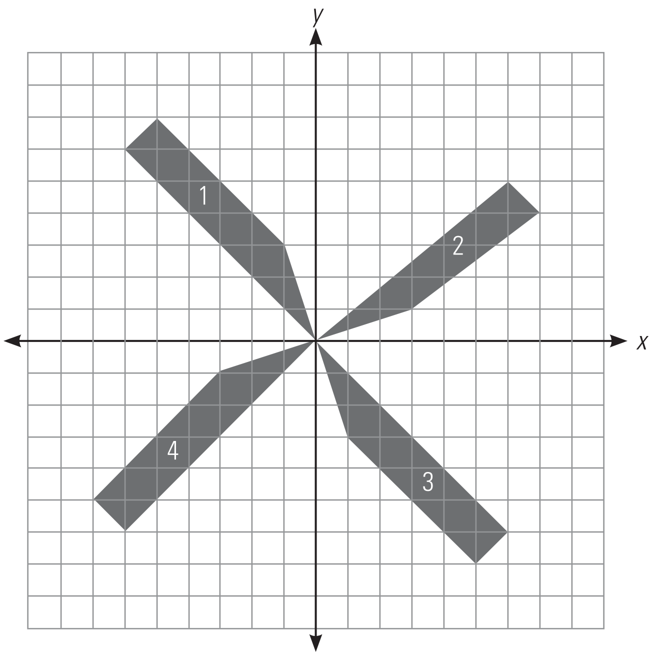
14. a) If a shape in the first quadrant is rotated 90° counter-clockwise about the origin, in what quadrant will the resulting image be located? How will the coordinates be affected?

b) Rotate shape ABC 90° counter-clockwise about the origin, and label the vertices A′, B′, and C′. State their coordinates.



c) Rotate shape ABC 270° clockwise about the origin on the same coordinate plane. How does the resulting image compare to A′B′C′? Why?

15. Misha is designing the propeller for a toy helicopter. The blades need to be evenly spaced around the origin (that is, three of the blades must be 90°, 180°, and 270° rotations of the first blade). Misha has drawn the following sketch of the propeller. If blade 1 is correctly placed, which of the other three blades is incorrectly drawn?



Tandem rotor helicopters have two sets of blades that rotate in opposite directions. The two blades increase the helicopter's lifting capacity, making the design popular for military use.

NEW SKILLS: WORKING WITH DILATIONS

A dilation is a transformation that enlarges or reduces a shape or an object. You need to know the **scale factor**, the number by which the side lengths are multiplied, in order to draw the dilation. A scale factor greater than 1 will result in an enlargement, and a scale factor of less than 1 will result in a reduction. The original shape and the image will be **similar figures**.

Although dilation is a commonly used term to mean making bigger, in math it is used in contexts with either enlargement or reduction.

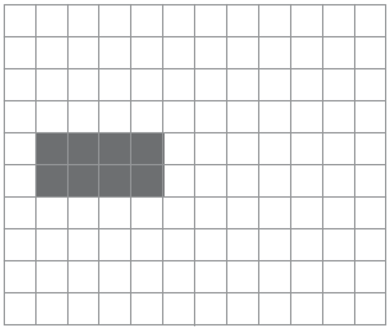
For more details, see page 224 of *MathWorks 12*.

scale factor: a number by which all the dimensions of an original figure are multiplied to produce an enlargement or reduction

similar figures: figures that are the same shape; their corresponding angles must be congruent and their corresponding sides must be proportional

Example 6

The rectangle below is 4 cm long and by 2 cm wide.



a) Dilate the rectangle by a scale factor of 2. What are the side lengths?

b) Dilate the rectangle by a scale factor of 0.5. What are the side lengths?

c) A dilation of the rectangle resulted in a rectangle 14 cm wide by 7 cm. What scale factor was used?

SOLUTION

a) To calculate the lengths, multiply each of the original dimensions by the scale factor.

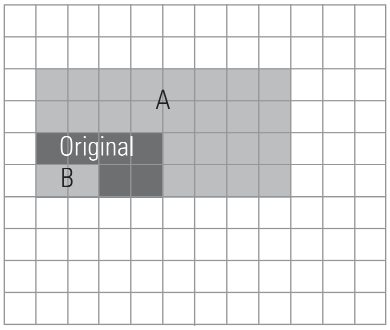
length: 2×4 cm = 8 cm
width: 2×2 cm = 4 cm

The dilated rectangle is 8 cm by 4 cm.

b) Multiply each of the original dimensions by the scale factor.

length: 0.5×4 cm = 2 cm
width: 0.5×2 cm = 1 cm

The dilated rectangle is 2 cm by 1 cm.



c) To calculate the scale factor, divide the dimensions of the dilation by the dimensions of the original.

length: 14 cm ÷ 4 cm = 3.5
width: 7 cm ÷ 2 cm = 3.5

The original rectangle was dilated by a scale factor of 3.5.

BUILD YOUR SKILLS

16. A scalene triangle has side lengths of 5.2 cm, 6.4 cm, and 9.6 cm. What will the lengths of the sides be if it undergoes a dilation by each of the following scale factors?

a) 7

b) 0.4

17. Rob took a digital photograph that is 9 inches by 15 inches.

a) He has one picture frame that is 5 inches by 8 inches, and one that is 8 inches by 10 inches. Can the photograph be dilated to fit the frames? Explain your answer.

b) Rob wants to make a poster of the photograph by dilating it by a scale factor of 3. What will the dimensions of the poster be, in feet and inches?

Do these gift boxes appear to be dilations of one another?

18. Emmanuelle is making three nesting wooden storage boxes in her woodworking shop. The mid-sized box is 45 cm long, 30 cm wide, and 25 cm tall.

    a) The larger box is 58.5 cm long. If it is a dilation of the mid-sized box, what scale factor was used? What are the width and height of the larger box?

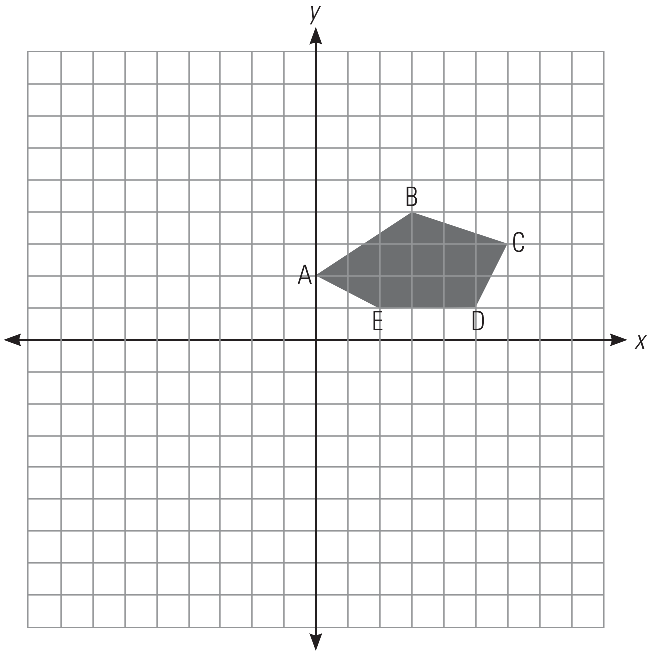
    b) The smaller box is a dilation of the mid-sized box, by a scale factor of $\frac{3}{4}$. What are the dimensions of the smaller box?

PRACTISE YOUR NEW SKILLS

1. Use pentagon ABCDE below to answer the following questions.

   a) Translate the shape 3 units to the left and 5 units down. Label the vertices A′, B′, C′, D′, and E′. What are the coordinates of the translated image?

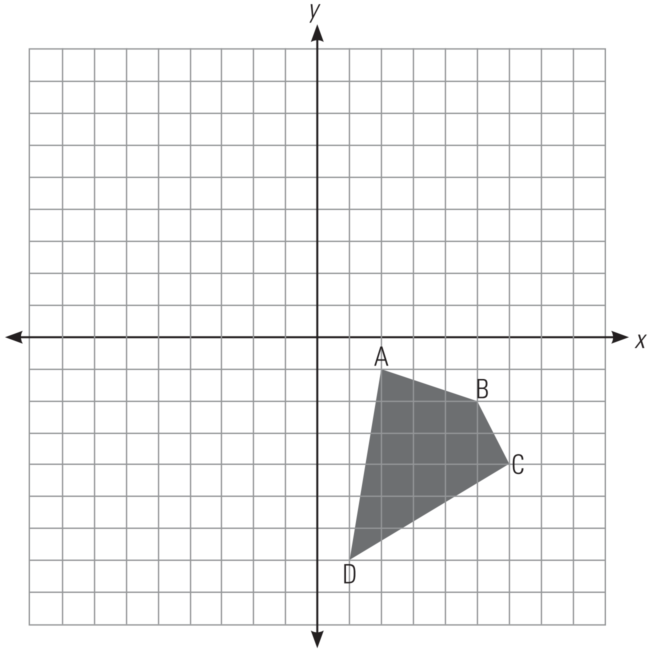
   b) Translate ABCDE so that vertex B moves to B″(–4, 5). Label the vertices A″, B″, C″, D″, and E″. What are the coordinates of the translated vertices? How far did ABCDE move left/right and up/down?



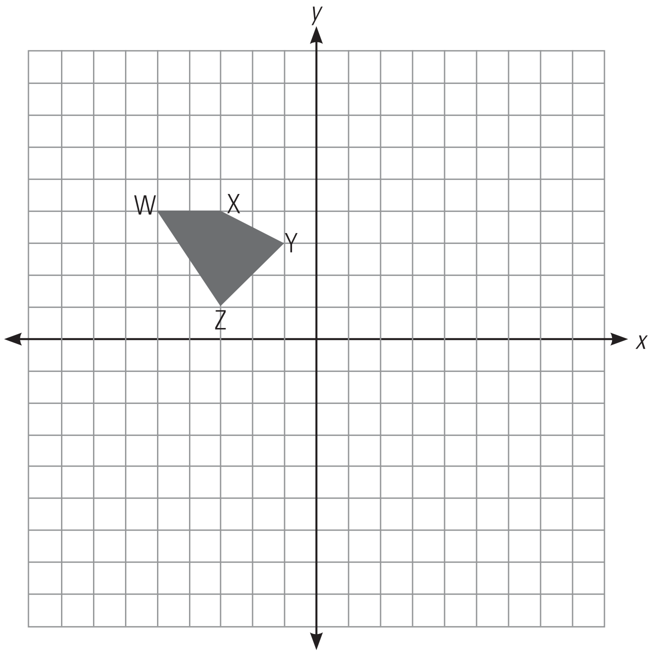
2. Use the following diagram to answer the following questions.

   a) Reflect the shape over the *x*-axis. Label its vertices as A′, B′, C′, and D′. State their coordinates

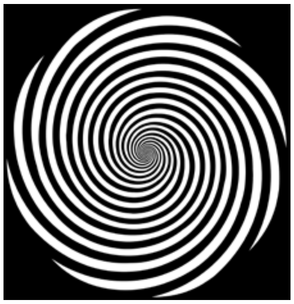
   b) Reflect ABCDE over the *y*-axis. Label its vertices as A″, B″, C″, and D″. State their coordinates.



3. Rotate the following image 270° clockwise about the origin. What are the coordinates of the vertices of the rotated image?



4. Examine the following images. Are they symmetrical or not? If the image is symmetrical, draw the line of symmetry or the reflection line.



5. Lenore is an architectural model builder. She creates scale models of buildings before they are built. The models are used to sell the building plans to builders and investors.

   Two common scales used for architectural models are $\frac{1}{100}$ and $\frac{1}{500}$.

   a) Would a model built in $\frac{1}{100}$ scale need to be enlarged or reduced to create a model in $\frac{1}{500}$ scale?

Building architectural models requires close attention to detail and a lot of math. Even trees have to be built to scale.

   b) Lenore is constructing a model of an apartment building. The completed building will be 145 metres tall. How tall would it be in $\frac{1}{100}$ scale? How tall would it be in $\frac{1}{500}$ scale?

   c) Lenore built another model of one of the floor plans of the new building. In the actual building, a hallway will be 8 m long. In Lenore's model, the hallway is 40 cm long. What scale did Lenore use?

6.2 Multiple Transformations

REVIEW: WORKING WITH PARALLEL LINES AND TRANSVERSALS

Often when a shape or object undergoes transformation, it undergoes more than one transformation. It may be rotated and then translated, for example, or reflected over the *y*-axis then translated.

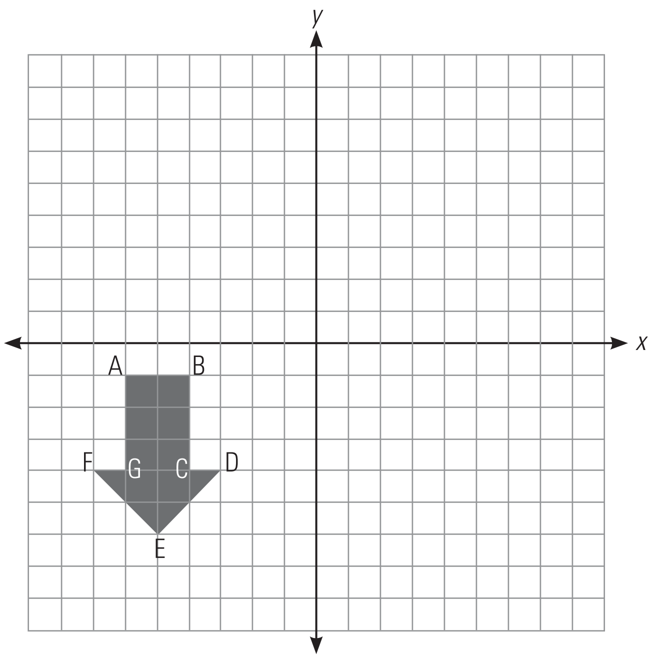
For more details, see page 231 of *MathWorks 12*.

Example 1

On the following coordinate plane, do the following transformations:

- Rotate the shape 90° clockwise about the origin, and draw the resulting image.
- Translate the rotated shape 4 units to the right and 3 units down, and draw the resulting image.

What are the coordinates of the vertices of the final image?



SOLUTION

Do the transformation in steps, completing the rotation before starting the translation.

The shape is first rotated 90° clockwise about the origin. The shape will move from the third quadrant to the second quadrant. For each vertex, the original *x*-coordinate will change signs, then the *x*- and *y*-coordinates will switch positions.

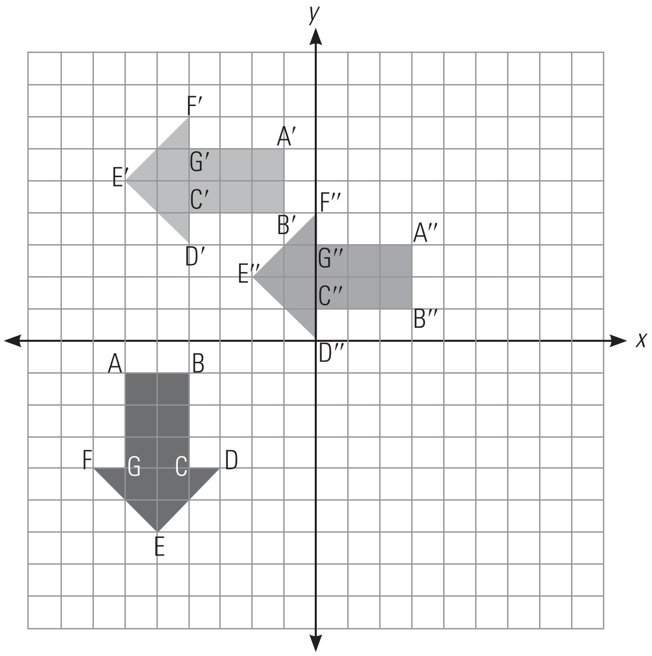
A(–6, –1) → A′(–1, 6)
B(–4, –1) → B′(–1, 4)
C(–4, –4) → C′(–4, 4)
D(–3, –4) → D′(–4, 3)
E(–5, –6) → E′(–6, 5)
F(–7, –4) → F′(–4, 7)
G(–6, –4) → G′(–4, 6)

Draw the rotated image on the coordinate plane as shown below.

Next, translate the shape 4 units to the right and 3 units down.

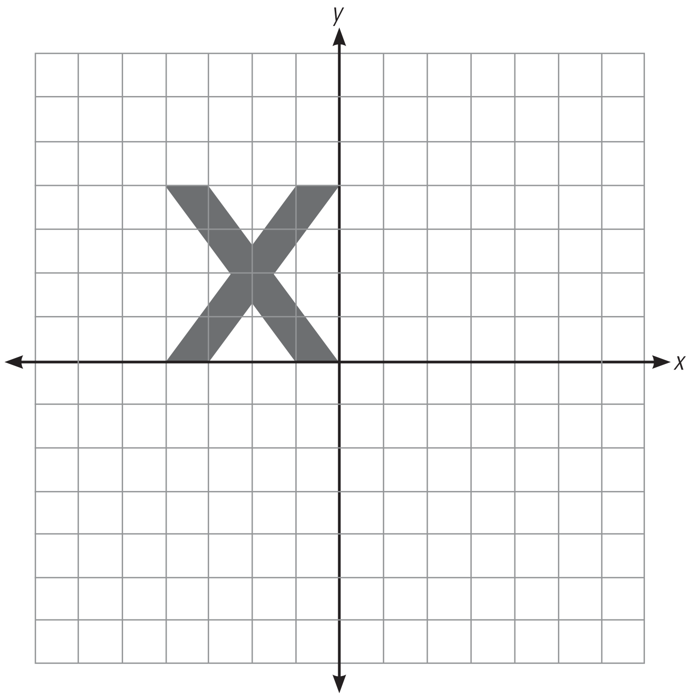
A′(–1, 6) → A″(3, 3)
B′(–1, 4) → B″(3, 1)
C′(–4, 4) → C″(0, 1)
D′(–4, 3) → D″(0, 0)
E′(–6, 5) → E″(–2, 2)
F′(–4, 7) → F″(0, 4)
G′(–4, 6) → G″(0, 3)

These are the final coordinates of the vertices.

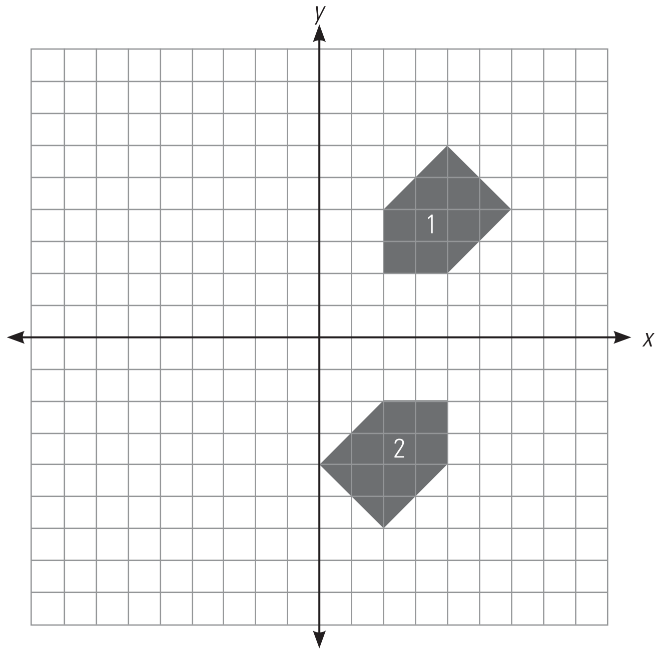


BUILD YOUR SKILLS

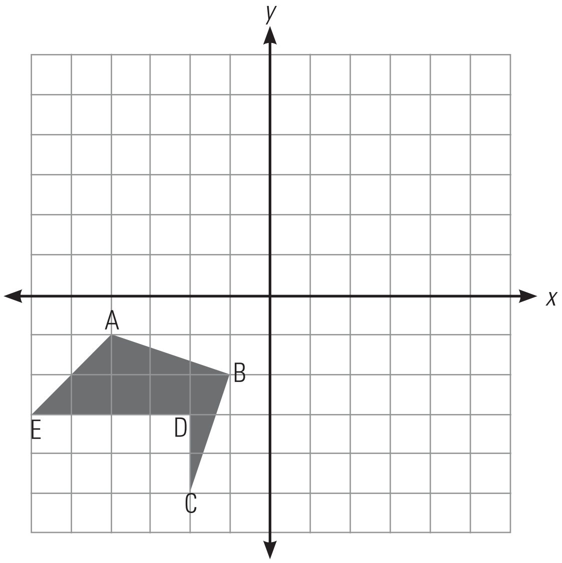
1. Rotate the following shape 270° clockwise about the origin, then translate it 2 units right and 3 units down. Show the intermediate step.



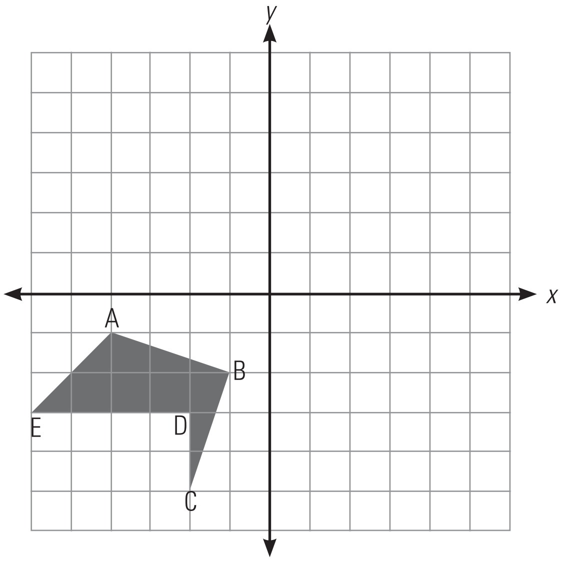
2. Shape 1 was rotated 180° clockwise about the origin, then translated to the position of shape 2. How far and in which direction(s) was the rotated image translated?



3. a) Rotate the following shape 90° clockwise about the origin, then translate it 6 units to the right.



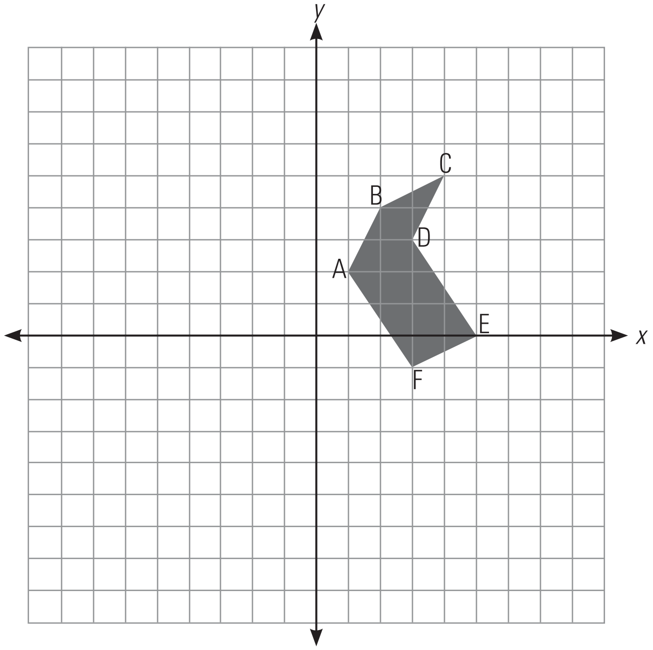
b) Translate the shape 6 units right, then rotate it 90° clockwise about the origin.



c) Are the final images in a) and b) the same? Why or why not?

Example 2

Reflect this shape over the *y*-axis, and then over the *x*-axis. What are the final coordinates of the vertices?



SOLUTION

Do the transformation in two steps, first completing the reflection over the *y*-axis, then doing the reflection over the *x*-axis.

When a shape is reflected over the *y*-axis, the *y*-coordinates stay the same and the *x*-coordinates change signs.

A(1, 2) → A′(–1, 2)

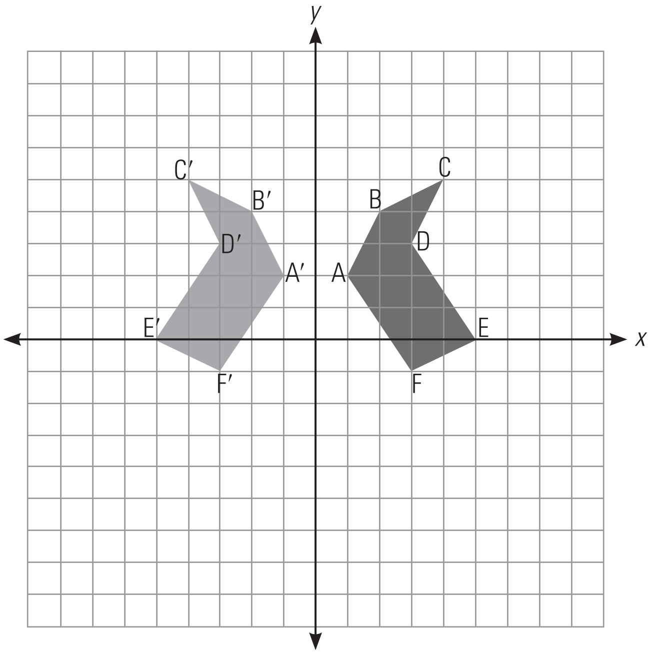
B(2, 4) → B′(–2, 4)

C(4, 5) → C′(–4, 5)

D(3, 3) → D′(–3, 3)

E(5, 0) → E′(–5, 0)

F(3, –1) → F′(–3, –1)



Next, reflect A′B′C′D′E′F′ over the *x*-axis. The *x*-coordinates stay the same and the *y*-coordinates change signs.

A′(–1, 2) → A″(–1, –2)

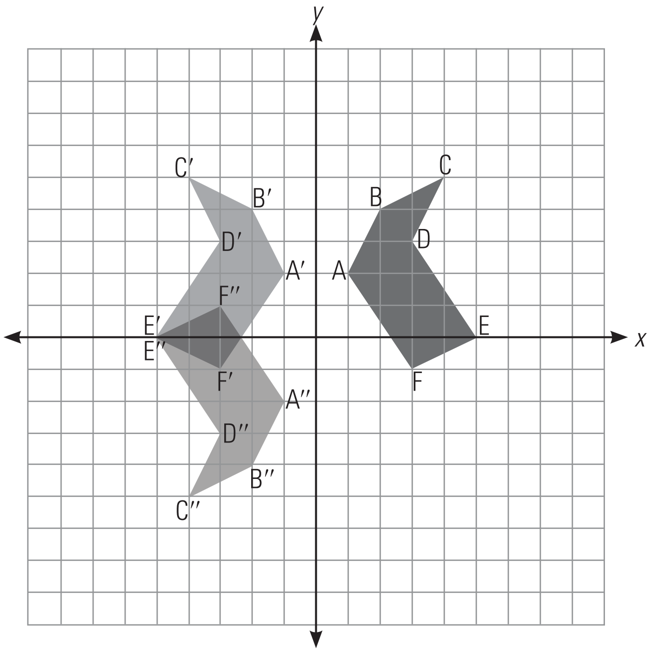
B′(–2, 4) → B″(–2, –4)

C′(–4, 5) → C″(–4, –5)

D′(–3, 3) → D″(–3, –3)

E′(–5, 0) → E″(–5, 0)

F′(–3, –1) → F″(–3, 1)



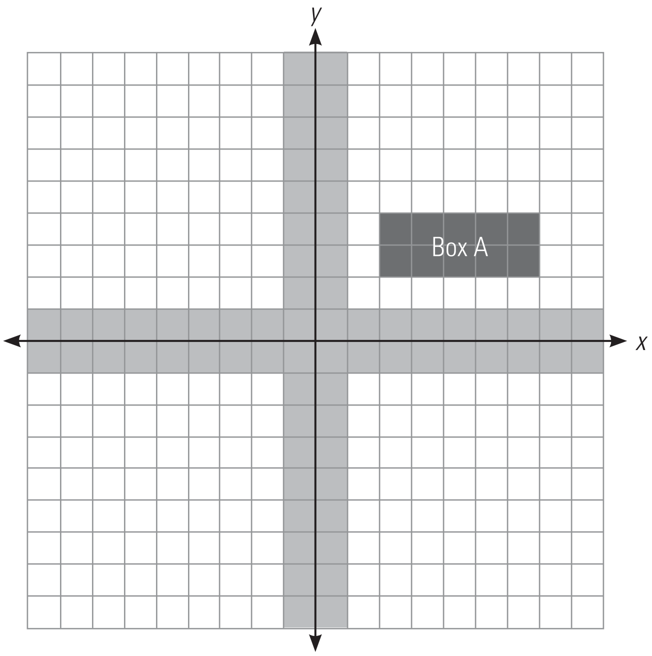
BUILD YOUR SKILLS

4. Elise is a landscape designer. She is designing a garden in a community park. She wants the flower beds to be symmetrically arranged. Box A is in the first quadrant of her design, and she wants to add boxes to the other three quadrants.

   a) Reflect box A over the *y*-axis, and label the resulting box "Box B."

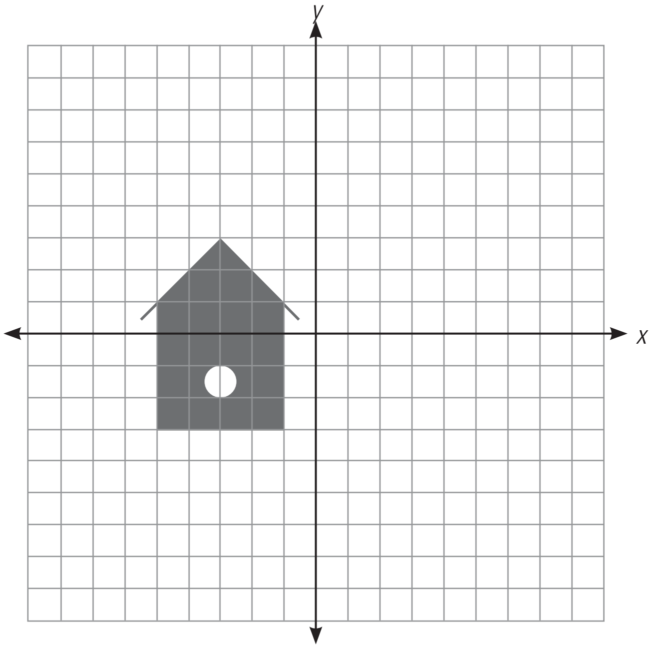
   b) Reflect box A over the *x*-axis, and label the resulting box "Box C."

   c) Reflect box C over the *y*-axis, and label the resulting box "Box D."

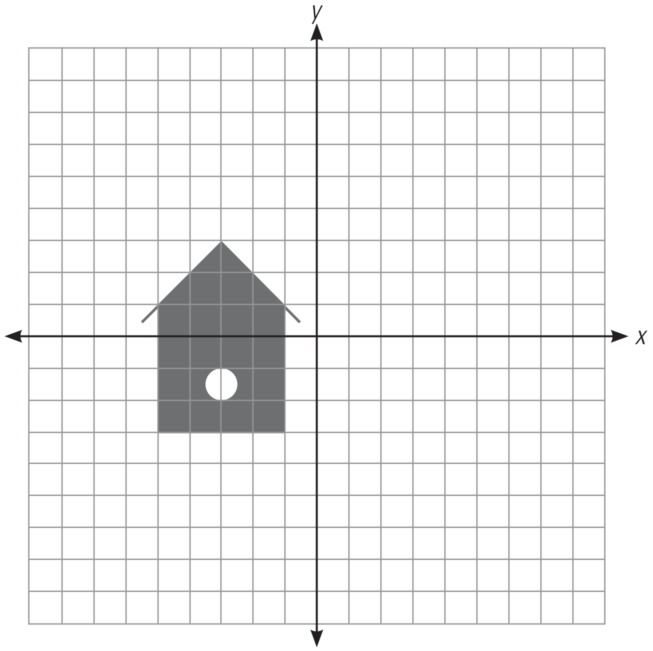


The vegetable gardens of the Château de Villandry in the Loire Valley of France are designed with many lines of symmetry. The Loire Valley is a UNESCO World Heritage Site, mainly due to its important architecture including the Château de Villandry, which was built in 1532. The castle's gardens have been landscaped to re-create the original designs of the 16th century.

5. a) Reflect the following shape over the *y*-axis, then translate it 4 units to the right and 2 units up.



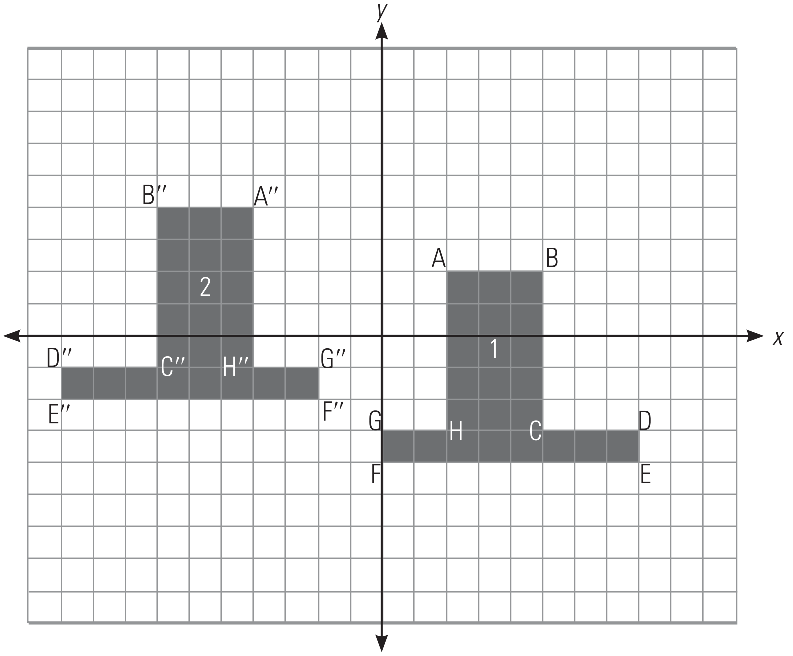
b) Translate the shape 4 units to the right and 2 units up, then reflect it over the *y*-axis.



c) Are the resulting images in a) and b) the same? Why or why not?

Example 3

Examine the diagram below. What transformations could have taken place to move from position 1 to position 2?



SOLUTION

The transformation appears to be a reflection over a vertical reflection line, followed by a transformation. If shape 1 were reflected over the *y*-axis, point A(2, 2) would move to A′(–2, 2). Calculate what translation would be required to move A′(–2, 2) to its final location of A″(–4, 4).

- It moves horizontally from –4 to –2, or 2 units to the left.
- It moves vertically from 2 to 4, or 2 units up.

The same translation, when applied to all vertices, results in shape 2.

ALTERNATIVE SOLUTION

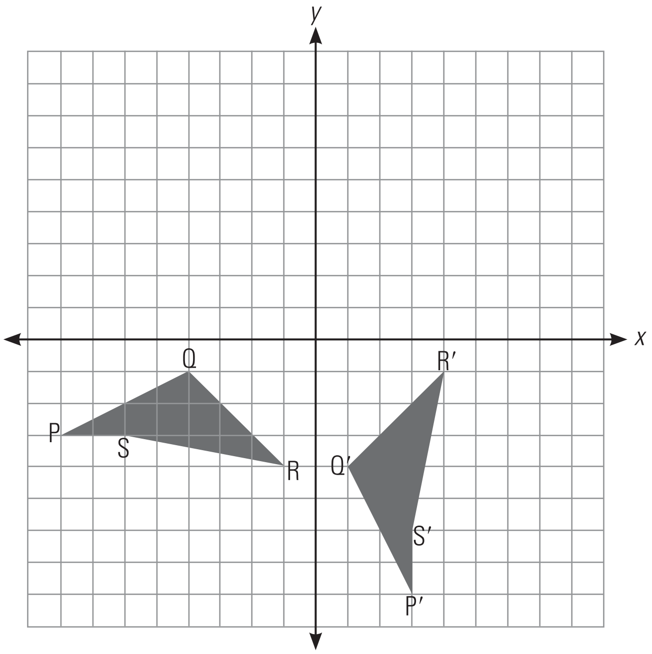
Shape 1 could have first been translated 2 units to the right and 2 units up, then reflected over the *y*-axis to produce shape 2.

Note: Other transformations may be possible to result in the same final image.

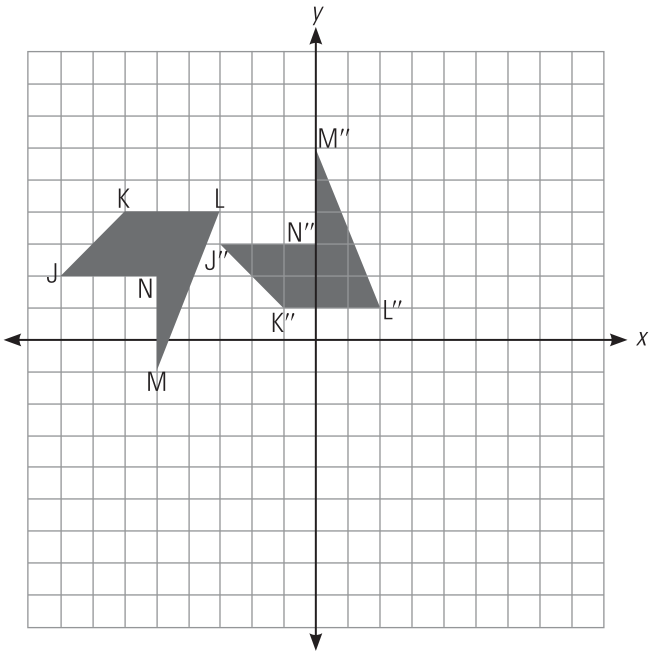
BUILD YOUR SKILLS

Stained glass was a popular art form in the Middle Ages, and was revived in the 19th century in Europe. Stained glass windows often incorporate patterns made of transformations. How many transformations can you identify in this design?

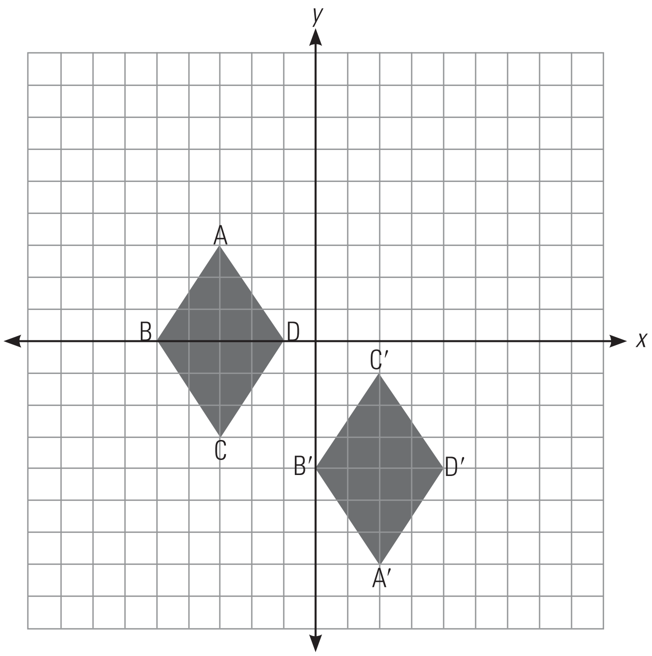
6. A single transformation (translation, reflection, or rotation) was used to move PQRS to P′Q′R′S′. Explain what transformation could have taken place.



7. The image J″K″L″M″N″ is the result of a translation followed by a reflection across a line. Explain what translation and reflection could have taken place.

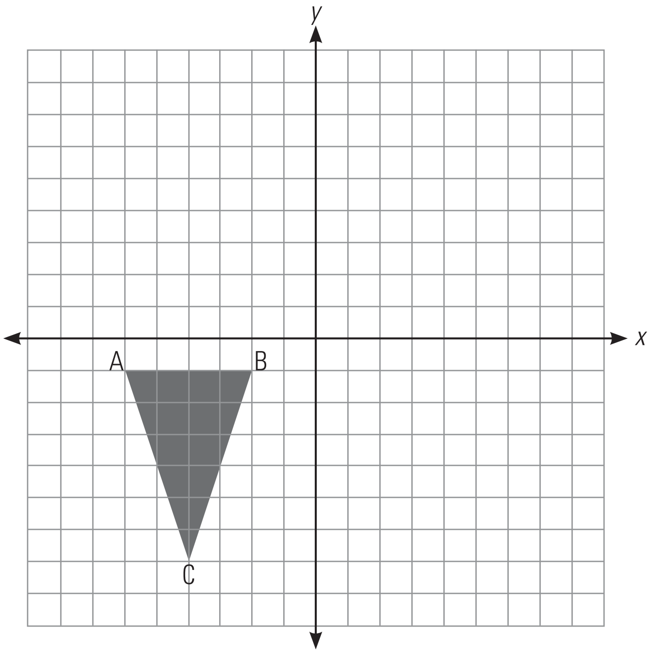


8. Hector says that image A′B′C′D′ is the result of a translation 5 units to the right and 4 units down of diagram ABCD. Camila disagrees; she says A′B′C′D′ was created by a reflection and translation. Who is right? Explain.



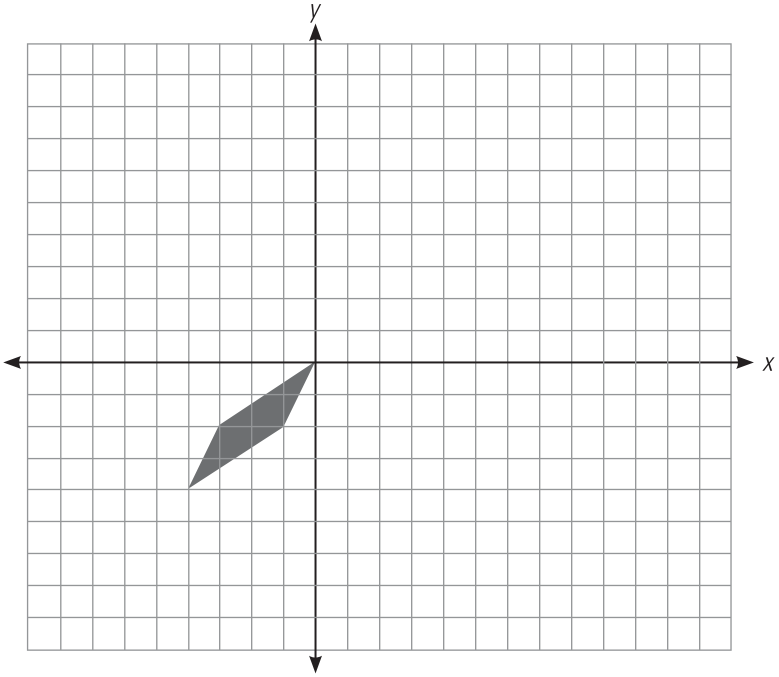
PRACTISE YOUR NEW SKILLS

1. Rotate the isosceles triangle 90° clockwise about the origin and label it A′B′C′. Next, reflect A′B′C′ over the *y*-axis and label it A″B″C″. State the coordinates.



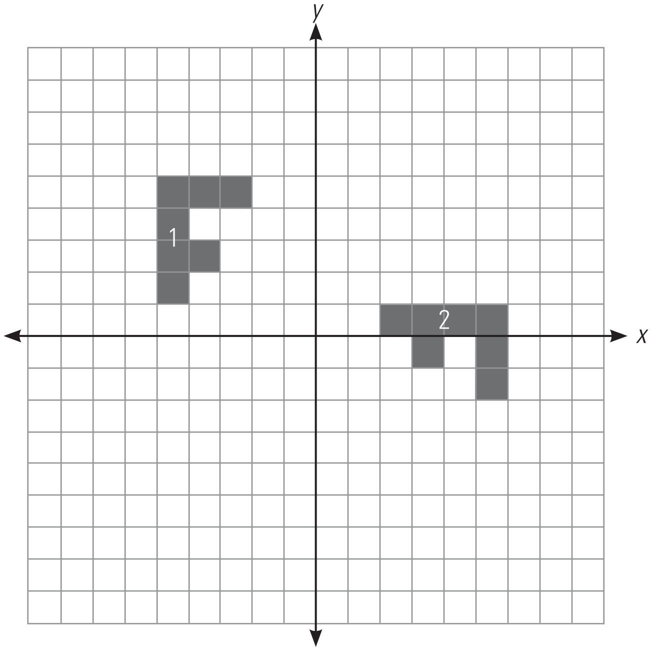
Choosing furniture upholstered with fabric in a bold pattern can enliven and brighten a room.

2. Layla is designing a pattern that will be printed on fabric. She wants to create a repeating pattern. She has drawn one section of the pattern. Rotate the image 90°, 180°, and 270° clockwise about the origin, then translate the entire image 8 units to the right and 4 units up.

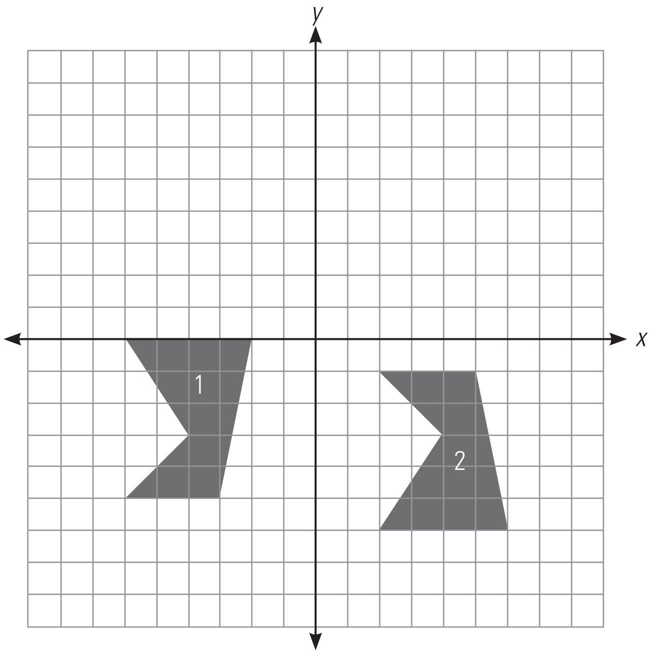


3. Examine the diagrams below. What specific transformations could have taken place to move the shapes from position 1 to position 2? Use the hints provided.

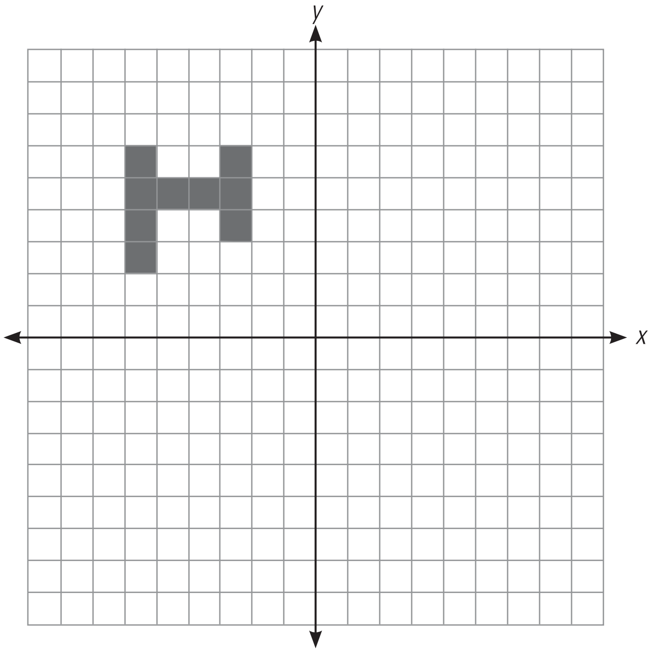
a) The transformation involves a rotation and a translation.



b) The transformation involves a translation and a reflection.



4. a) Translate the following shape 2 units left and 3 down, then reflect it over the *y*-axis.



b) If you reflect the original shape over the *y*-axis first, what translation will be necessary to create the same image as in a)?

CHAPTER TEST

Are the blades of this windmill perfect rotations about a centre point?

1. a) A point A(–5, 3) is translated 3 units right and 4 units down. What are its new coordinates?

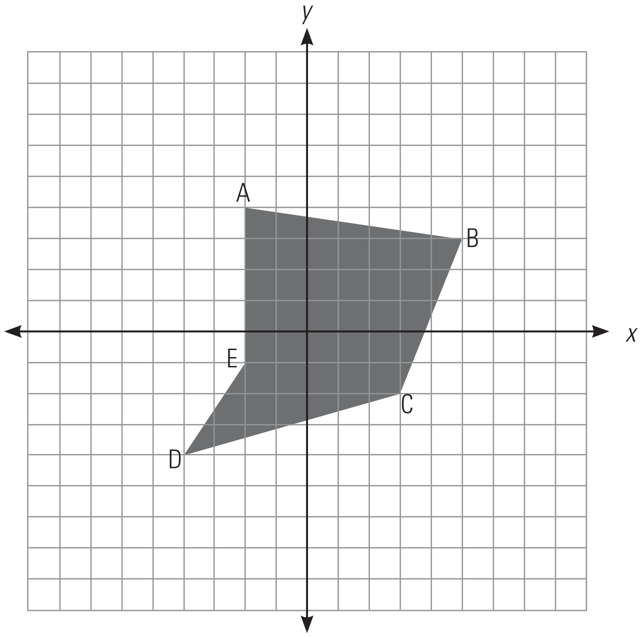
   b) A point C(–5, 3) is reflected across the *y*-axis. What are its new coordinates?

2. A point B(7, 4) is rotated 90° clockwise about the origin.

   a) In which quadrant is point B′?

   b) What are the coordinates of point B′?

3. Translate the following pentagon so that vertex A(–2, 4) moves to A′(1, 0).



   a) How far did each vertex move, and in which direction(s)?

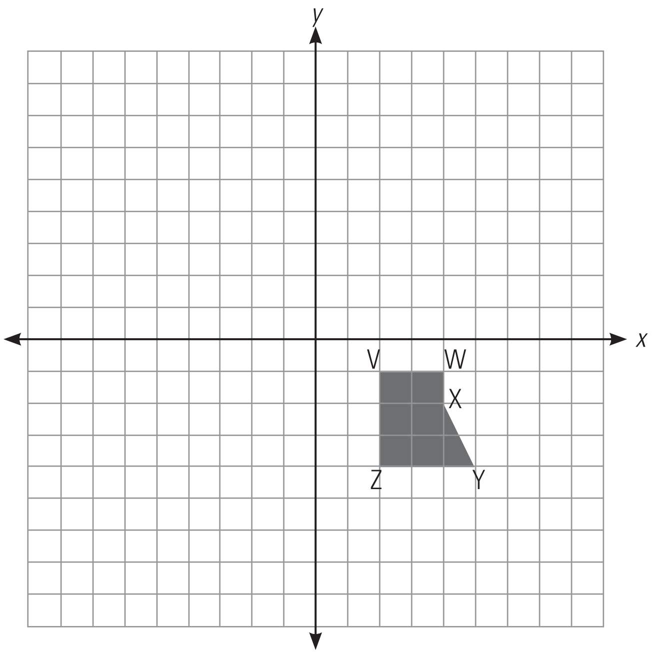
b) What are the coordinates of B′, C′, D′, and E′?

4. Rotate the following diagram:

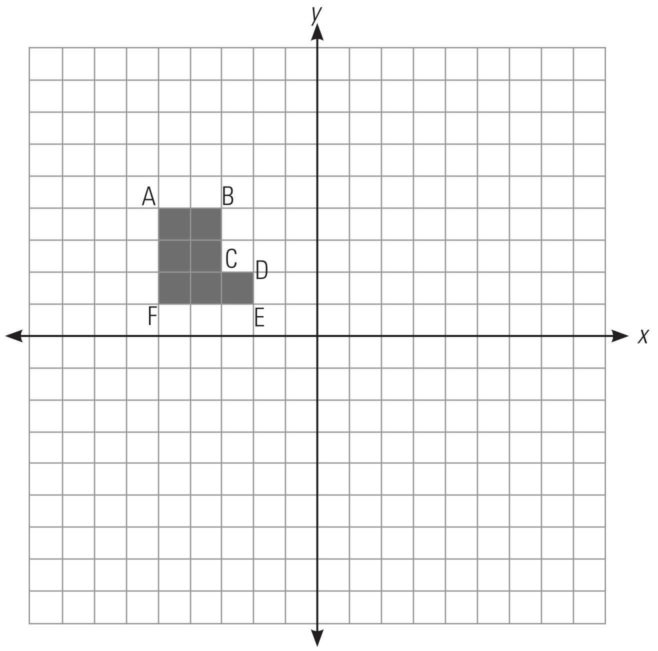
a) 90° clockwise about the origin. Label its vertices V′, W′, X′, Y′, and Z′, and state their coordinates.

b) 180° clockwise about the origin. Label its vertices V″, W″, X″, Y″, and Z″, and state their coordinates.

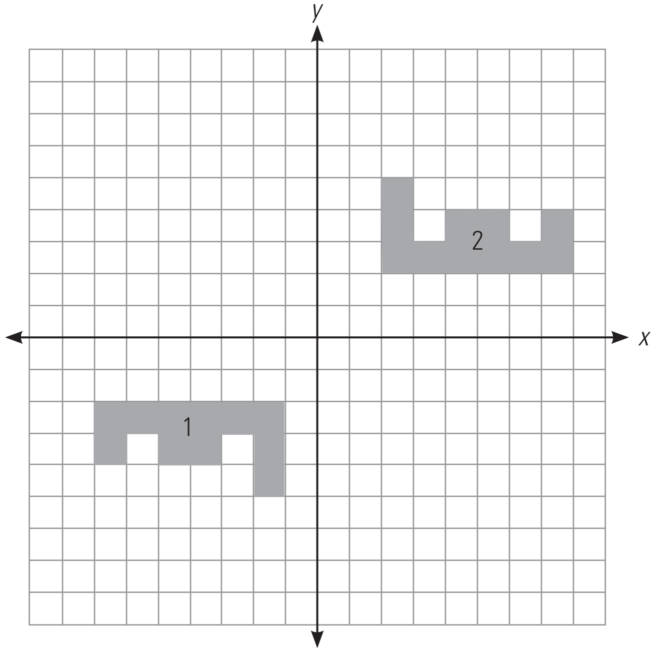
c) Reflect V″W″X″Y″Z″ over the *y*-axis. Label its vertices ABCDE, and state their coordinates.



5. Rotate the following shape 270° about the origin, then translate it 5 units to the right and 3 units up. What are the coordinates of the vertices of the resulting image?

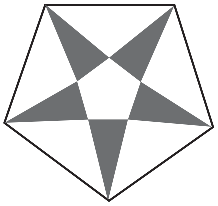


6. Shape 1 underwent transformations to become shape 2. What transformation(s) could have been applied to create the image?

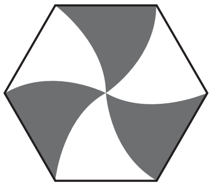


7. How many lines of symmetry does each image have? Draw in the line(s) of symmetry, if they exist.

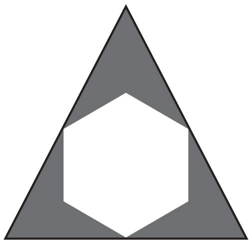
a)



b)



c)



d)

8. Katya creates handmade sets of nesting dolls, which she sells to craft and souvenir stores. One set includes five dolls. The mid-sized doll has a middle diameter of 7 cm and a height of 12 cm.

Nesting dolls were first designed and crafted in Russia, where they are called Matryoshka dolls. In the traditional design, the outermost doll shows a woman in traditional Russian peasant dress, the inner dolls depict either boys or girls, and the smallest doll is a baby. Matryoshka designs can now be found showing everything from movie stars, to politicians, to athletes.

a) The largest doll has a diameter of 12 cm. If it is a dilation of the mid-sized doll, what scale factor was used? What is the height of the largest doll?

b) The smallest doll is a dilation of the mid-sized doll, by a scale factor of 0.3. What are the diameter and height of the smallest doll?

c) Katya made another set of dolls. In the new set, the largest doll has a diameter of 8.75 cm and a height of 15.6 cm. Is this a dilation of the first set of dolls?

Chapter 7
Trigonometry

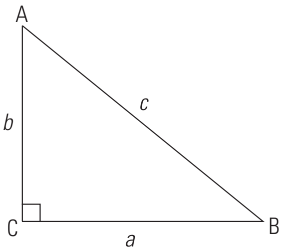
Surveyors use a transit or theodolite to measure horizontal and vertical angles, allowing them to calculate distances.

7.1 The Sine Law

REVIEW: WORKING WITH TRIGONOMETRIC RATIOS

In the past, you have worked with the trigonometric ratios of sine, cosine, and tangent to solve for unknown side lengths and angle measures. The ratios are defined as:

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



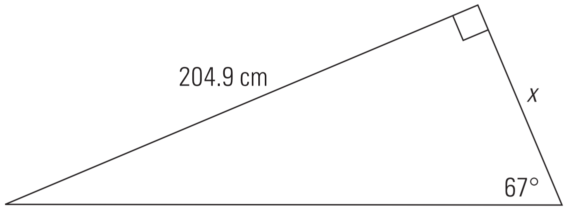
These ratios can be written as follows for ∠A in triangle ABC:

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$

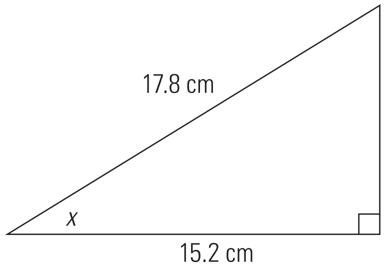
Example 1

Calculate the value of x.

a)



b)



SOLUTION

> Unless otherwise indicated, solve for side lengths to the nearest tenth of a unit and angle measures to the nearest degree.

a) You are given the length of the side opposite the given angle, and you need to solve for the side adjacent to the angle. Use the tangent ratio.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 67^\circ = \frac{204.9}{x}$$

$$x\tan 67^\circ = \frac{204.9}{x} \times x$$

$$x\tan 67^\circ = 204.9$$

$$\frac{x\tan 67^\circ}{\tan 67^\circ} = \frac{204.9}{\tan 67^\circ}$$

$$x = \frac{204.9}{\tan 67^\circ}$$

$$x \approx 87.0 \text{ cm}$$

b) You are given the side adjacent to the unknown angle, and the hypotenuse. Use the cosine ratio.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos x = \frac{15.2}{17.8}$$

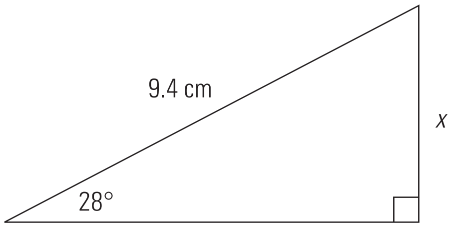
$$x = \cos^{-1}\left(\frac{15.2}{17.8}\right)$$

$$x \approx 31^\circ$$

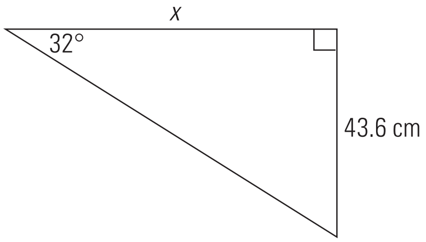
BUILD YOUR SKILLS

1. Calculate the length of the indicated side.

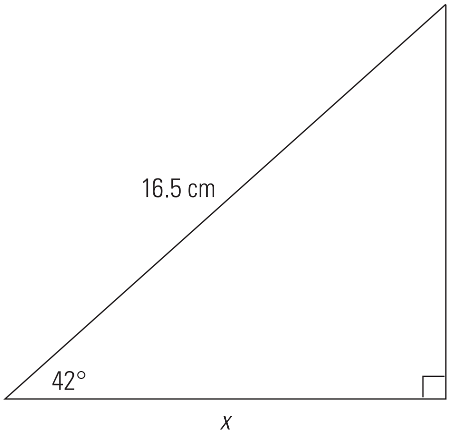
a)



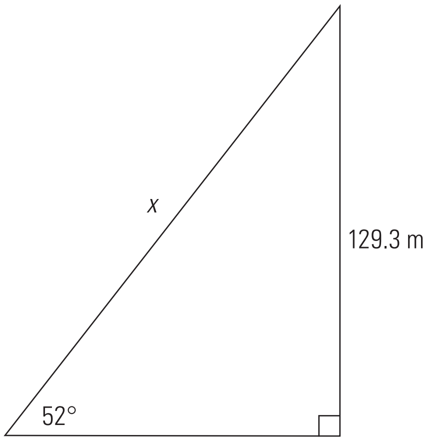
b)



c)

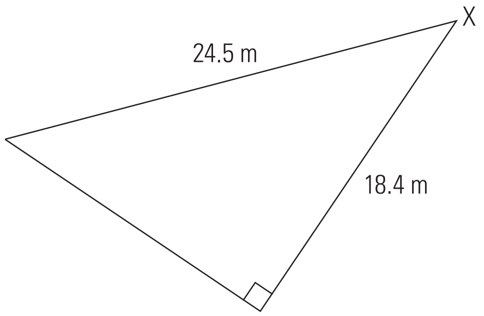


d)

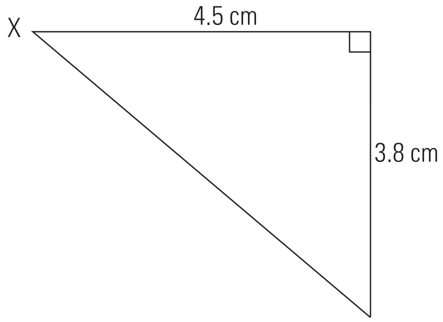


2. Calculate the size of angle X.

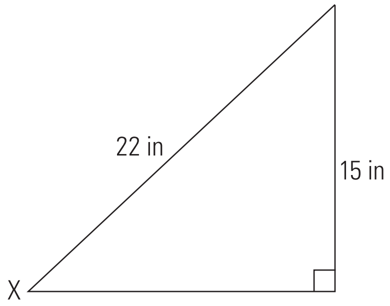
a)



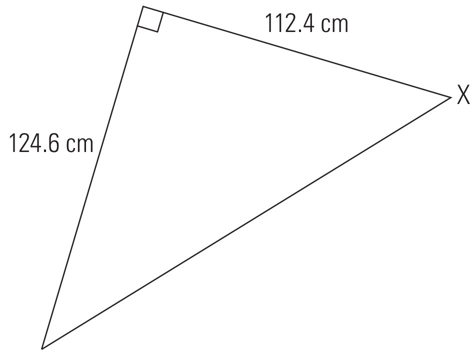
b)



c)



d)



NEW SKILLS: WORKING WITH THE SINE LAW

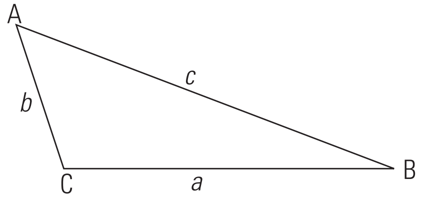
oblique triangle: a triangle with no angle equal to 90°

sine law: for all triangles, the ratio of an angle to the side opposite is equal to the ratio for all angle/opposite side pairs

Using the trigonometric ratios of sine, cosine, and tangent, as well as the Pythagorean theorem, you can solve for missing side lengths or angle measures of right triangles. But what happens if you want to know a missing measurement from an **oblique triangle** (a triangle without a right angle)?

The **sine law** can be used to solve for an unknown side length or an unknown angle measure of an oblique triangle in certain situations. The sine law states that, for a triangle ABC:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



If you know two angles of a triangle, you also know the third one because the sum of the angles of a triangle is always 180°.

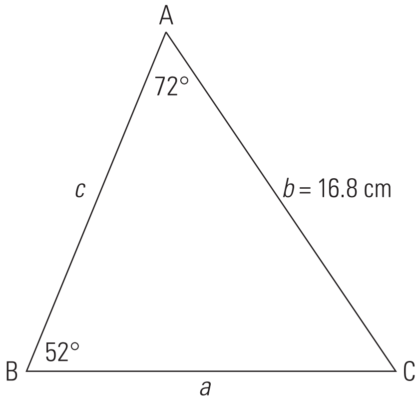
It can be used when you know:

- two sides and the angle opposite one of them. This would allow you to solve for the angle opposite the other known side. For example, if you know sides *a* and *b* and ∠A, you can solve for ∠B.
- two angles and the side opposite one of them. This would allow you to solve for the side opposite the other known angle. For example, if you know ∠A, ∠B, and side *a*, you can solve for *b*.

For more details, see page 249 of *MathWorks 12*.

Example 2

Use the sine law to calculate the length of side *a*.



SOLUTION

You are given ∠A and ∠B, as well as side *b*. Use the sine law to solve for *a*.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 72°}{a} = \frac{\sin 52°}{16.8}$$

$$a \times \frac{\sin 72°}{a} = \frac{\sin 52°}{16.8} \times a$$

$$\sin 72° = \frac{a \sin 52°}{16.8}$$

$$16.8 \times \sin 72° = \frac{a \sin 52°}{16.8} \times 16.8$$

$$16.8 \sin 72° = a \sin 52°$$

$$\frac{16.8 \sin 72°}{\sin 52°} = \frac{a \sin 52°}{\sin 52°}$$

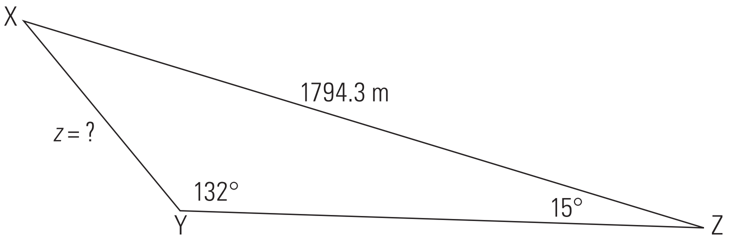
$$\frac{16.8 \sin 72°}{\sin 52°} = a$$

$$20.3 \text{ cm} \approx a$$

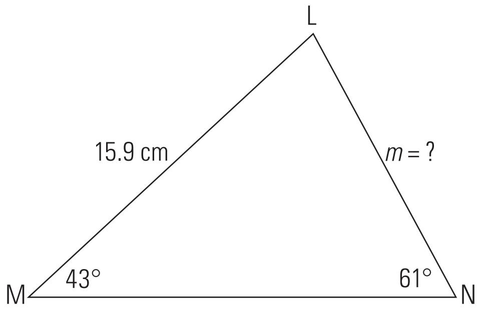
BUILD YOUR SKILLS

3. Calculate the length of the indicated side.

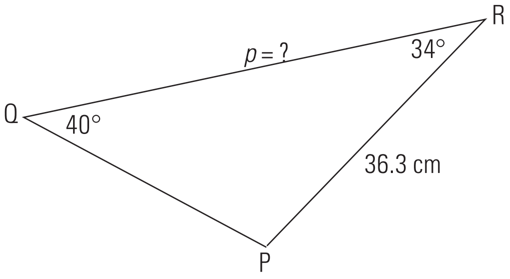
a)



b)

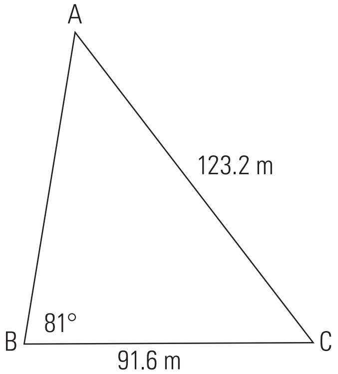


c)



Example 3

Calculate the sizes of ∠A and ∠C.



SOLUTION

You are given side lengths *a* and *b*, and ∠B. With this information, you can use the sine law to calculate ∠A.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{91.6} = \frac{\sin 81°}{123.2}$$

$$91.6 \times \frac{\sin A}{91.6} = \frac{\sin 81°}{123.2} \times 91.6$$

$$\sin A = \frac{91.6 \sin 81°}{123.2}$$

$$A = \sin^{-1}\left(\frac{91.6 \sin 81°}{123.2}\right)$$

$$A \approx 47°$$

Now calculate the measure of ∠C.

$$180° = \angle A + \angle B + \angle C$$

$$180° = 47° + 81° + \angle C$$

$$180° - 47° - 81° = \angle C$$

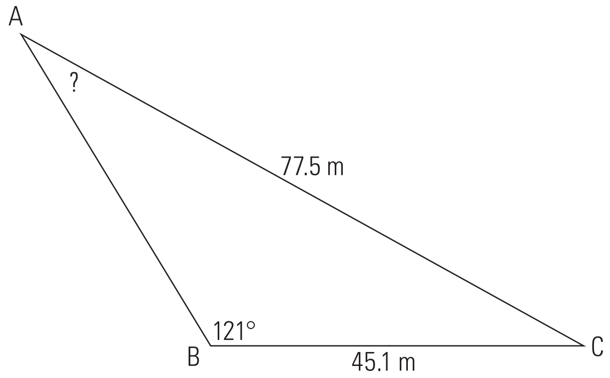
$$52° = \angle C$$

The triangle is a musical instrument usually made of steel and bent into an equilateral triangle. Early versions of this instrument had an isosceles shape.

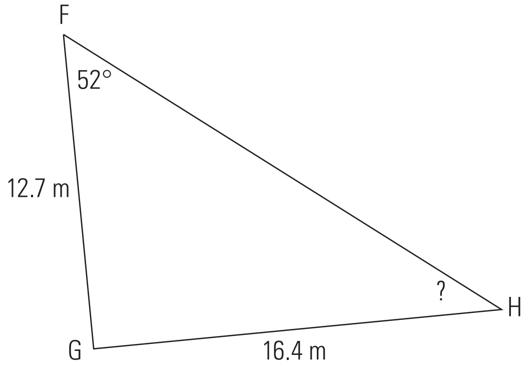
BUILD YOUR SKILLS

4. Calculate the measure of the indicated side and/or angle.

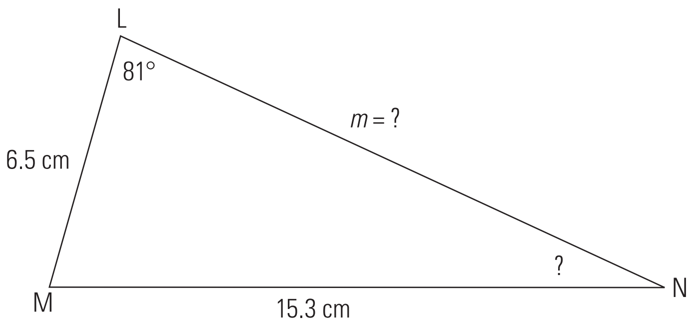
a)



b)



c)

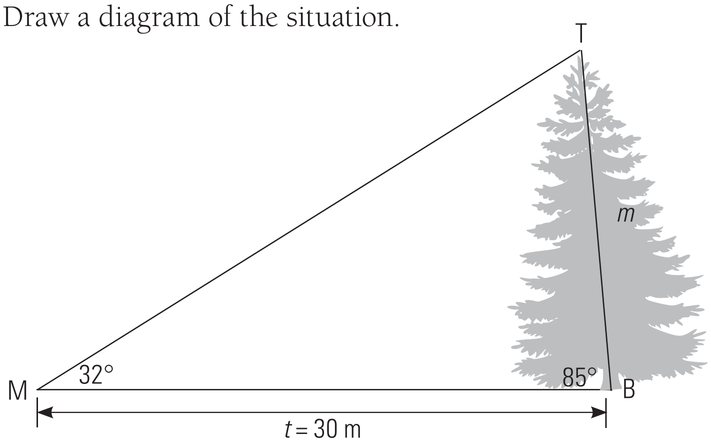


Example 4

Miguel is an arborist and he needs to calculate the height of a tree that he will be removing from a construction site. The tree is leaning at an angle of 85° with the ground. Miguel stands 30 m from the base of the tree and calculates the angle of elevation to the top of the tree to be 32°. What is the height of the tree?

SOLUTION

Draw a diagram of the situation.



You are given ∠M, ∠B, and *t*. This is not enough information to use the sine law. Calculate ∠T.

$$180° = \angle M + \angle B + \angle T$$
$$180° = 32° + 85° + \angle T$$
$$180° - 32° - 85° = \angle T$$
$$63° = \angle T$$

Now use the sine law to calculate *m*.

$$\frac{\sin M}{m} = \frac{\sin T}{t}$$
$$\frac{\sin 32°}{m} = \frac{\sin 63°}{30}$$
$$m \times \frac{\sin 32°}{m} = \frac{\sin 63°}{30} \times m$$
$$\sin 32° = \frac{m \sin 63°}{30}$$
$$30 \times \sin 32° = \frac{m \sin 63°}{30} \times 30$$
$$30 \sin 32° = m \sin 63°$$

Tree trimming can be a dangerous occupation if not done carefully. It requires training and a lot of safety equipment.

$$\frac{30 \sin 32°}{\sin 63°} = \frac{m \sin 63°}{\sin 63°}$$

$$\frac{30 \sin 32°}{\sin 63°} = m$$

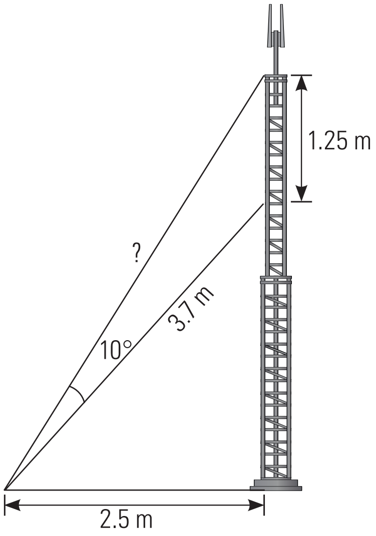
$$17.8 \text{ m} \approx m$$

The tree is approximately 17.8 m tall.

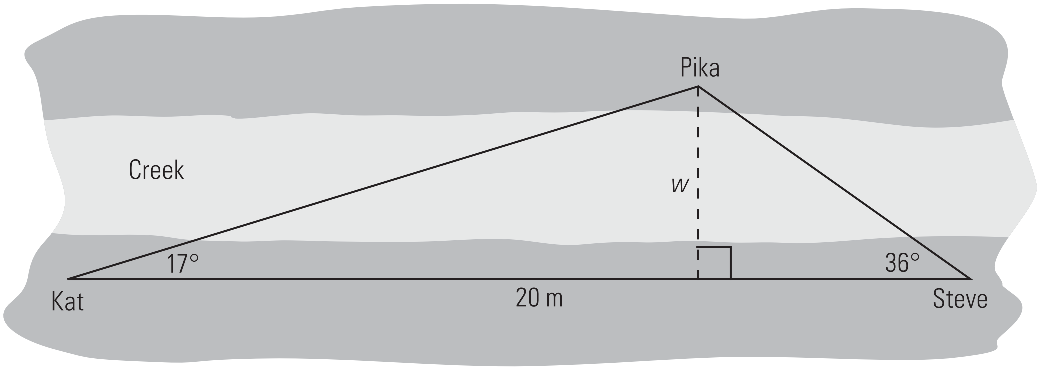
BUILD YOUR SKILLS

Steel lattice is the most widely used form of construction used for the creation of towers. The triangular cross-section is the most commonly used design.

5. A communications tower is stabilized with guy wires. One wire is 3.7 m long. A second guy wire is attached to the pole 1.25 m higher than the first wire, from the same anchor point on the ground. The angle made between the two wires is 10°. What is the length of the second wire?



6. Kat and Steve are hiking in Bugaboo Provincial Park in BC. In Silver Basin Meadow, they follow a creek. They are 20 m apart on one side of the creek when they spot a pika (a small mammal) on the opposite side of the creek.



Bugaboo Provincial Park is situated in the Purcell Mountain range of southeast BC. People come from all over the world to hike in the region and climb the granite spires of the mountains.

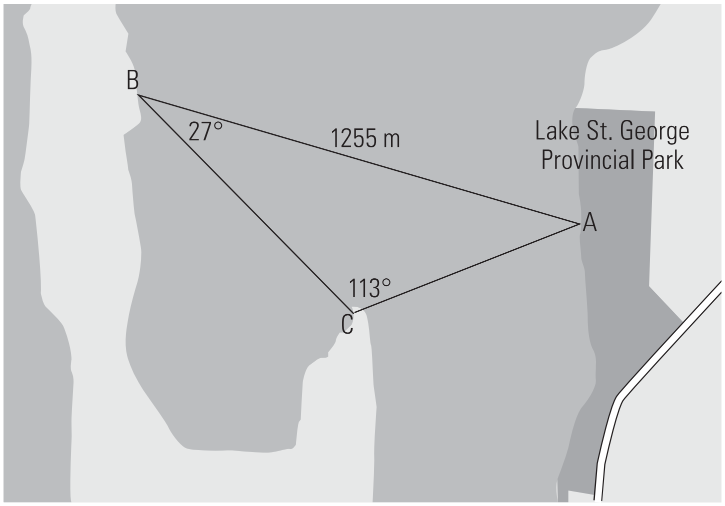
a) How far is Kat from the pika?

b) How far is Steve from the pika?

c) How wide is the creek?

Lake St. George is a popular camping and fishing spot. The lake is home to walleye, northern pike, and perch.

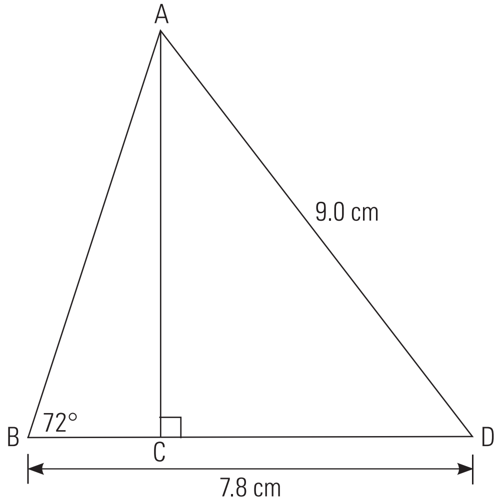
7. Alexander is camping at Lake St. George Provincial Park in Manitoba. He has taken a row boat out onto the lake. How far will he have to row if he travels from point A to point B, to point C, then back to point A?



Example 5

Kareem is sewing a sail for a toy boat. The sail will be made of two right triangles.

He has cut out the material for ΔABC, and ∠B is 72°. He knows that side AD must be 9.0 cm long, and the base of the finished sail must be 7.8 cm. At what angle must he cut ∠DAC?



SOLUTION

Calculate the measure of ∠DAB using the sine law.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin \angle DAB}{BD} = \frac{\sin B}{AD}$$

$$\frac{\sin \angle DAB}{7.8} = \frac{\sin 72°}{9.0}$$

$$7.8 \times \frac{\sin \angle DAB}{7.8} = \frac{\sin 72°}{9.0} \times 7.8$$

$$\sin \angle DAB = \frac{7.8 \sin 72°}{9.0}$$

$$\angle DAB = \sin^{-1}\left(\frac{7.8 \sin 72°}{9.0}\right)$$

$$\angle DAB \approx 56°$$

Next, calculate the measure of ∠D.

∠D = 180° – ∠DAB – ∠B

∠D = 180° – 56° – 72°

∠D = 52°

Finally, calculate the measure of ∠DAC.

∠DAC = 180° – ∠D – ∠DCA

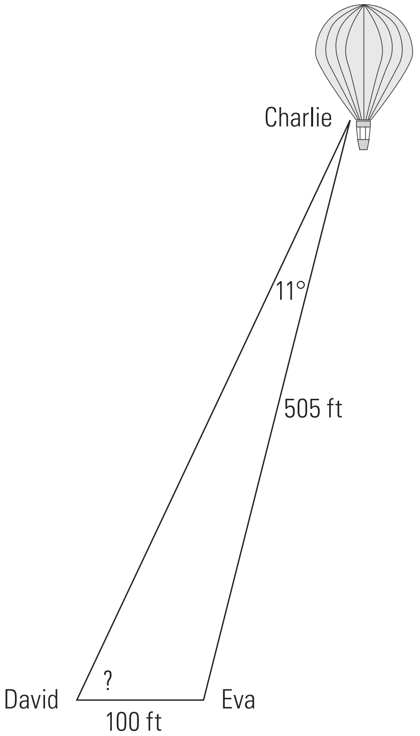
∠DAC = 180° – 52° – 90°

∠DAC = 38°

Kareem needs to cut ∠DAC at an angle of 38°.

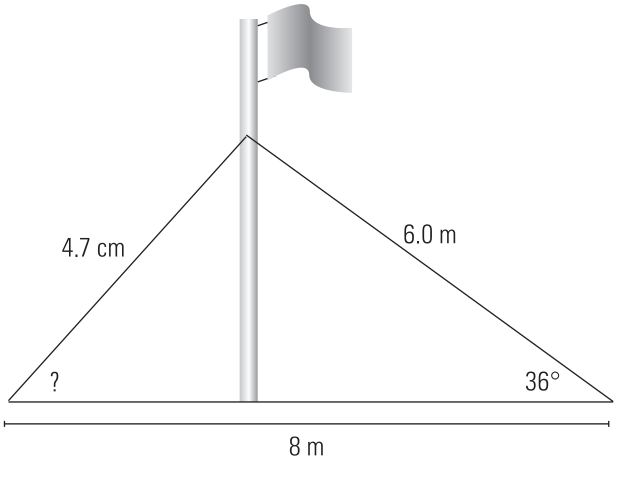
Building a toy boat, from a kit or from scratch, can be a fun way to explore boat designs and materials.

BUILD YOUR SKILLS



8. Charlie is in a hot air balloon above Regina, SK. His friends David and Eva are watching from the ground. David and Eva are standing 100 ft apart, and the angle between Charlie's line of sight to the two of them is 11°. Eva is 505 ft from the balloon. What is the angle of elevation of David's line of sight to the balloon?

9. A flag pole is stabilized using two guy wires. The wires are attached to the ground on opposite sides of the flag pole, 8 m apart. The wire on the left is 4.7 m long. The wire on the right is 6.0 m long and is attached at an angle of elevation of 36°.



a) What is the angle of elevation of the wire on the left?

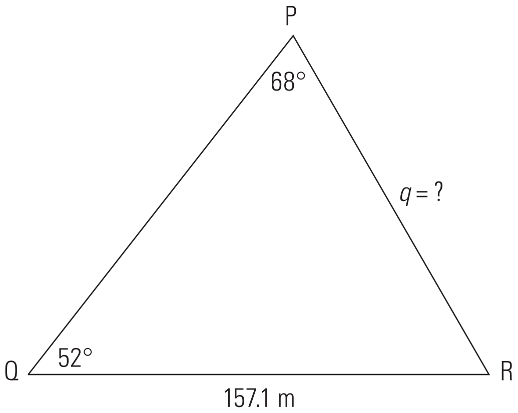
b) How high up the flag pole are the wires attached?

PRACTISE YOUR NEW SKILLS

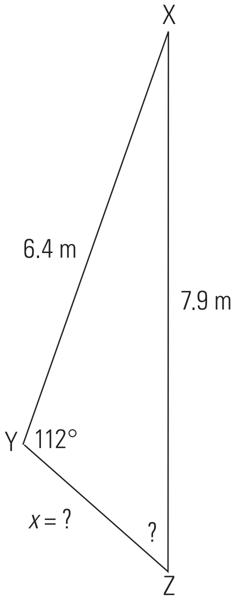
1. Calculate the measure of the indicated side and/or angle.

Hang gliders use a triangle control frame system to steer while in flight.

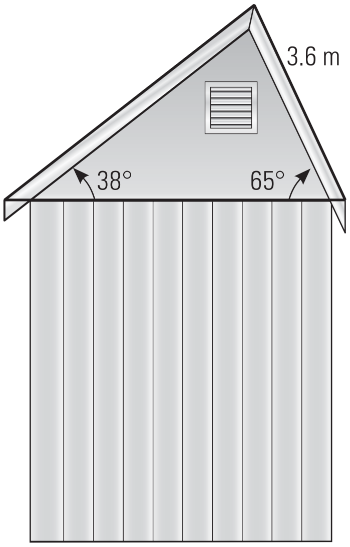
a)



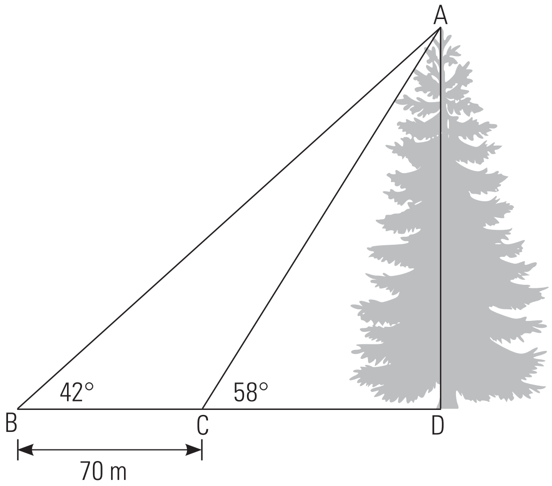
b)



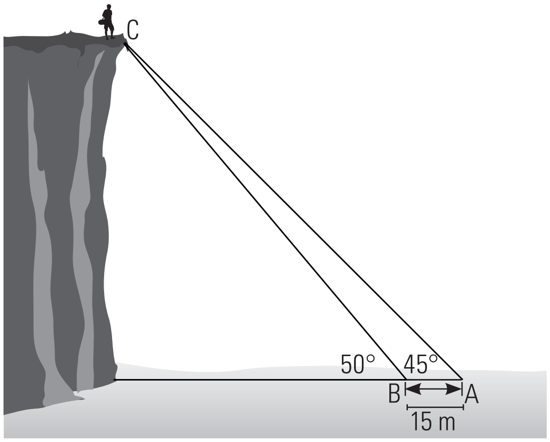
2. The roof-line of a ski chalet is asymmetrical. The shorter side is 3.6 metres long and makes an angle of 65° with the horizontal. The longer side makes an angle of 38° with the horizontal. How wide is the chalet?



3. The angle of elevation to the top of a tree from point B is 42°. From point C, which is 70 m closer to the tree, the angle of elevation to the top of the tree is 58°. How tall is the tree?



4. Aidan and Belinda are hiking when they notice people rock-climbing on a cliff nearby. They want to estimate the height of the cliff. Aidan estimates the angle of elevation to the top of the cliff from his location is about 45°. Belinda is 15 m closer to the cliff, and she estimates the angle of elevation to be 50°.



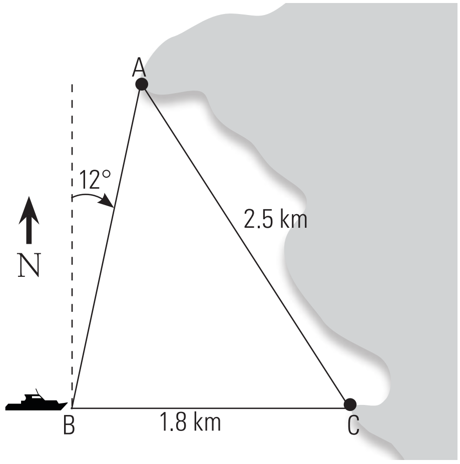
a) What is the distance from Aidan's location to the top of the cliff?

b) What is the height of the cliff?

c) How far from the base of the cliff is Aidan standing?

The Stawamus Chief in Squamish, BC, is a popular spot for hikers and rock climbers. It is more than 700 m tall and is considered a place of spiritual significance to the Squamish people, an indigenous group.

5. The captain of a boat sees two points of land: point C is 1.8 km east of the boat and point A is at a bearing of 12°. If points A and C are 2.5 km apart, how far is the boat from point A?



The Cosine Law 7.2

NEW SKILLS: WORKING WITH THE COSINE LAW

In section 7.1, you used the sine law to solve for unknown side lengths or angles of oblique triangles, but only if you knew two side lengths and an angle opposite one of them, or two angles and the side length opposite one of them.

What happens if you want to know a side length or angle in other situations? There is another trigonometric law that you can use, the **cosine law.**

cosine law: a relation between the sides of a triangle and the cosine of one of its angles

With the cosine law, you can solve for:

- the third side length of a triangle if you know two side lengths and the angle between them; or
- any angle if you know the lengths of the three sides of the triangle.

The letters in the formula are placeholders—they can be replaced with any letters. Just make sure the side you put in the *a* position is opposite the angle you put in the ∠A position.

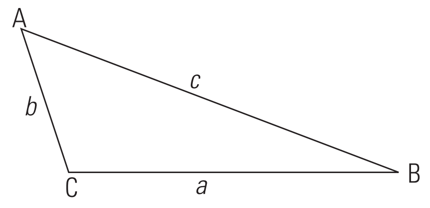
The cosine law states that, in a triangle ABC:

$$a^2 = b^2 + c^2 - 2bc \cos \text{A}$$

This can also be written as:

$$\cos \text{A} = \frac{b^2 + c^2 - a^2}{2bc}$$

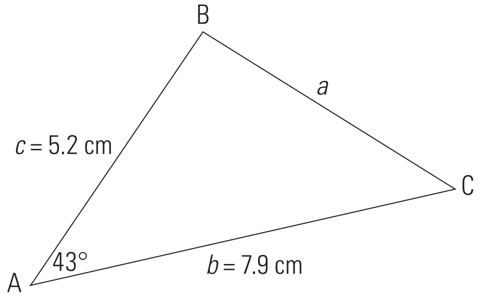
Note that these are just alternate ways of writing the same formula.



For more details, see page 260 of *MathWorks 12*.

Example 1

Calculate the length of side *a*.



SOLUTION

You know two side lengths, *b* and *c*, and the angle between them, ∠A. You can use the cosine law to calculate the length of side *a*.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = (7.9)^2 + (5.2)^2 - 2(7.9)(5.2) \cos 43°$$

$$a^2 = 62.41 + 27.04 - 82.16 \cos 43°$$

$$a^2 = 89.45 - 82.16 \cos 43°$$

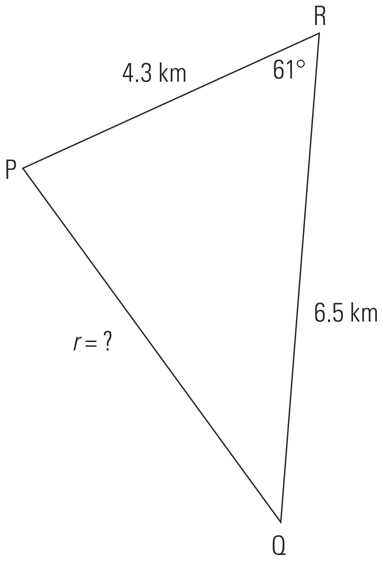
$$a = \sqrt{89.45 - 82.16 \cos 43°}$$

$$a \approx 5.4 \text{ cm}$$

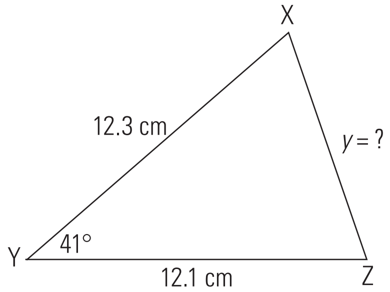
BUILD YOUR SKILLS

1. Calculate the length of the indicated side.

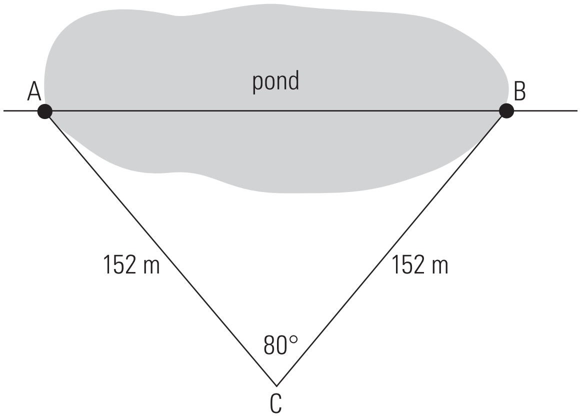
a)



b)

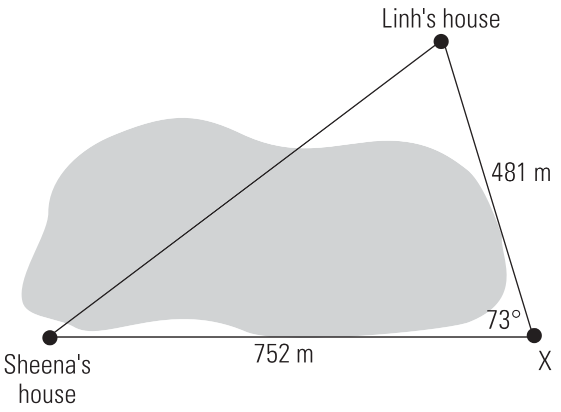


2. Brent is a surveyor. He has been hired by city planners who are developing a park within the city. They want to build a bridge over a pond, so that a walking trail can be installed. Brent collects the following measurements. What is the length of the pond from A to B?



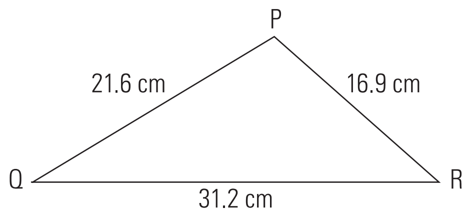
A surveyor conducting a geodetic survey, a land survey that takes into account the curvature of the earth.

3. Sheena and Linh live on opposite sides of a pond. To get to Linh's house, Sheena has to follow a sidewalk to the end of the pond (point X), then take a path through the park to Linh's house, as shown in the following diagram. How much farther does Sheena have to walk than if she could take a straight-line path to Linh's house?



Example 2

Calculate the size of ∠Q.



Note that this formula is the same as the one given in Example 1, but it has been rearranged to solve for the angle.

SOLUTION

You are given the three side lengths, but no angles. You can use the cosine law to solve for any of the angles. Use this form of the cosine law formula:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Substitute in the variables of the ΔPQR, with Q taking the place of A.

$$\cos Q = \frac{p^2 + r^2 - q^2}{2pr}$$

$$\cos Q = \frac{(31.2)^2 + (21.6)^2 - (16.9)^2}{2(31.2)(21.6)}$$

$$\cos Q = \frac{973.44 + 466.56 - 285.61}{1347.84}$$

$$\cos Q = \frac{1154.39}{1347.84}$$

$$Q = \cos^{-1}\left(\frac{1154.39}{1347.84}\right)$$

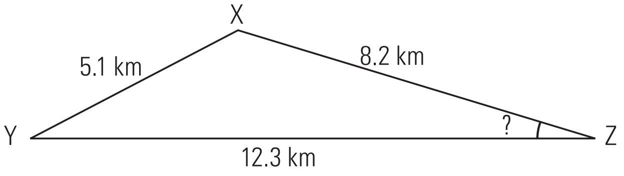
$$Q \approx 31°$$

The measure of ∠Q is approximately 31°.

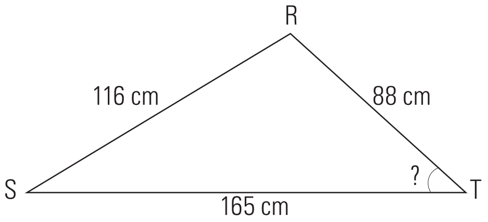
BUILD YOUR SKILLS

4. Calculate the size of the indicated angle.

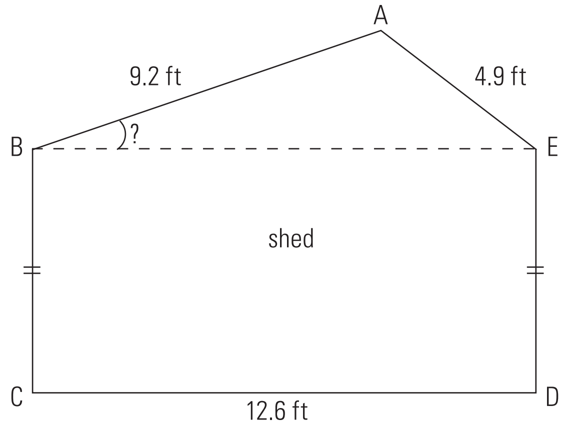
a)



b)



5. At what angle of elevation is the longer section of this shed roof installed?



6. Giles has three pieces of wood that he wants to use to build a frame for his flower garden. The pieces are 10.2 feet, 7.5 feet, and 8.4 feet long.

   a) Draw and label a diagram showing the triangular garden frame.

   b) What will be the measures of the angles at the corners of the garden frame?

Example 3

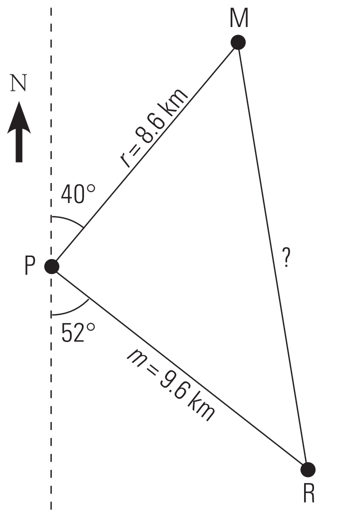
Mariya and Raphael are both going hiking through a park that has fairly level ground. They both leave from the same spot in the parking lot. Mariya follows one trail, walking 4.3 km/h in a direction 40° east of north. Raphael follows a different trail, walking 4.8 km/h in a direction 52° east of south. How far apart are they after 2 hours?

SOLUTION

Calculate how far they have each walked in 2 hours.

Mariya: 2 h × 4.3 km/h = 8.6 km
Raphael: 2 h × 4.8 km/h = 9.6 km

Draw a diagram showing their locations after 2 hours.



The north-south line is straight, so you can calculate ∠P of the triangle.

$\angle P = 180° - 40° - 52°$
$\angle P = 88°$

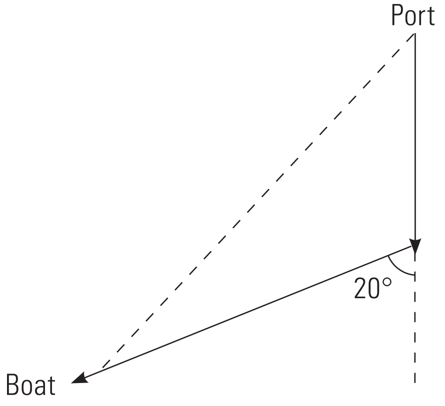
Use the cosine law to solve for the third side length.

$$p^2 = m^2 + r^2 - mr \cos P$$
$$p^2 = (9.6)^2 + (8.6)^2 - 2(9.6)(8.6) \cos 88°$$
$$p^2 = 92.16 + 73.96 - 165.12 \cos 88°$$
$$p^2 = 166.12 - 165.12 \cos 88°$$
$$p = \sqrt{166.12 - 165.12 \cos 88°}$$
$$p \approx 12.7 \text{ km}$$

After 2 hours, Mariya and Raphael are 12.7 km apart.

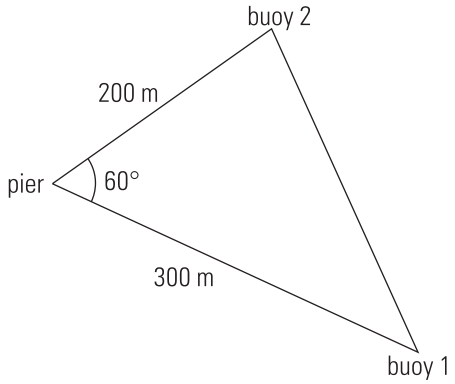
BUILD YOUR SKILLS

7. A ship travels south from port for 2 hours at 35 km/h. It then changes course and travels 20° west of south for 3 hours at the same speed. How far is it from port after 5 hours?



8. Gurjinder is a beach lifeguard. She must set up the buoys for a swimming competition. Swimmers will jump off the pier, swim around one buoy, change direction and swim around a second buoy, then head back to the pier. The first buoy is 300 m from the pier, and the second is 200 m from the pier. The angle between them is 60°. How long is the swim route?

For more than 100 years, the Lifesaving Society of Canada has been providing lifesaving programs in all provinces and territories in Canada.

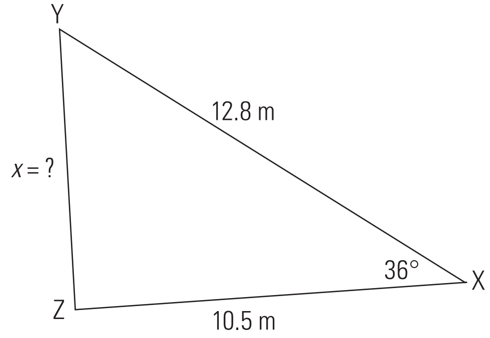


9. On the triangular frame of a bicycle, the distance from the seat (S) to the handlebars (H) is 52 cm, the handlebars to the gears (G) is 56 cm, and the gears to the seat is 42 cm. What is the angle formed at S?

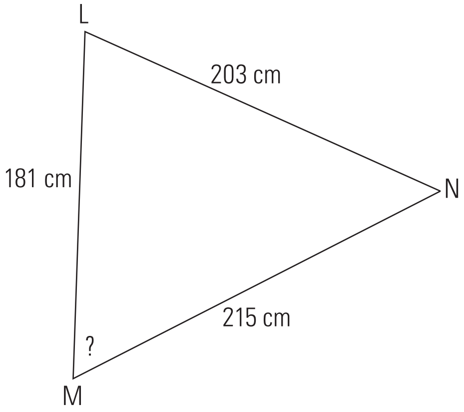
PRACTISE YOUR NEW SKILLS

1. Calculate the measure of the indicated side or angle.

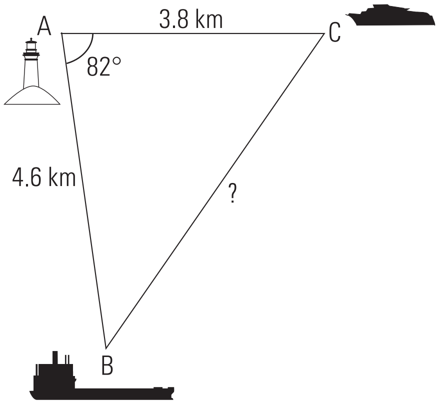
   a)



   b)

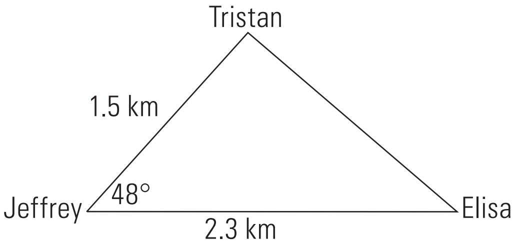


2. A lighthouse operator observes two ships at distances of 4.6 km and 3.8 km. The angle between the lines of observation is 82°. How far apart are the two ships?



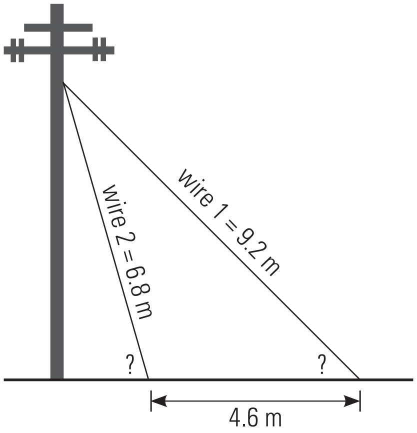
Peggys Point Lighthouse marks the eastern entrance of St. Margarets Bay in Nova Scotia. It is one of the most-photographed structures in Atlantic Canada.

3. From Jeffrey's house, the angle between the paths to Tristan's and Elisa's houses is 48°. It is 1.5 km to Tristan's house and 2.3 km to Elisa's. Jeffrey walks from his house to Tristan's and picks him up. They then walk to Elisa's, pick her up, and all go back to Jeffrey's house. How far will Jeffrey walk in total?



A ship's mast is supported by shrouds and stays—the nautical equivalents of guy wires

4. A radio tower is stabilized with guy wires. Two wires are attached at the same point on one side of the tower and are affixed to the ground 4.6 m apart. The first wire is 9.2 m long and the second is 6.8 m long.



a) At what angle of elevation is wire 1 installed?

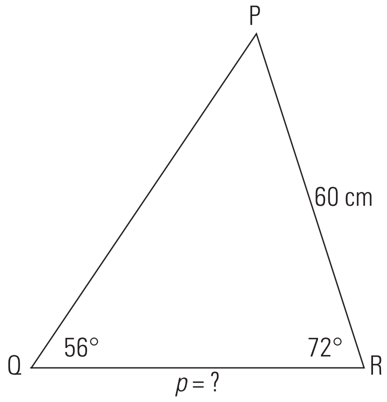
b) How high up the tower are the wires attached?

c) How far from the base of the tower is wire 1 attached to the ground?

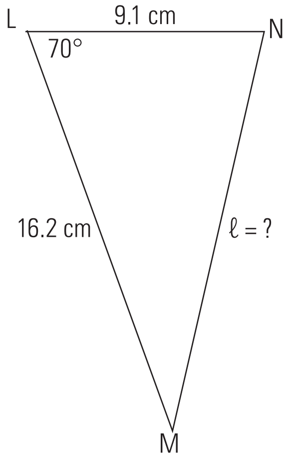
CHAPTER TEST

1. Calculate the length of the indicated side.

a)

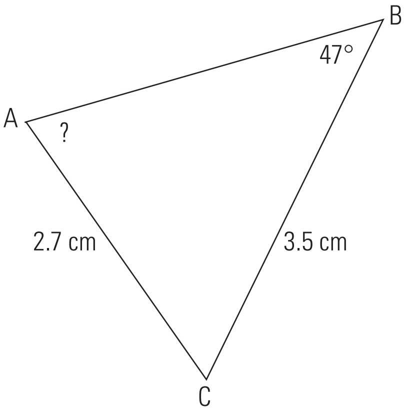


b)

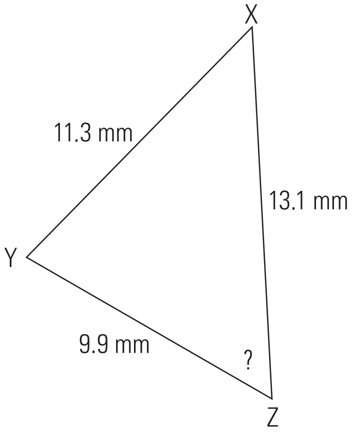


2. Calculate the size of the indicated angle.

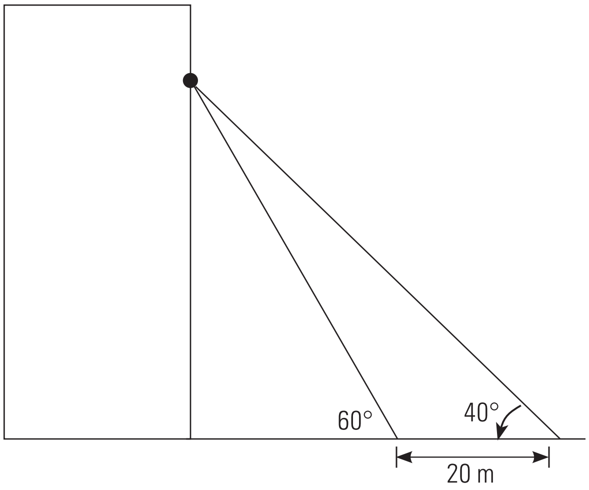
a)



b)



3. A contractor needs to repair a gargoyle on a building. At a distance away from the base of the building, he estimates that the angle of elevation to the gargoyle is 40°. He then moves 20 m closer to the base of the building and estimates the angle of elevation to be 60°. How high above the ground is the gargoyle?

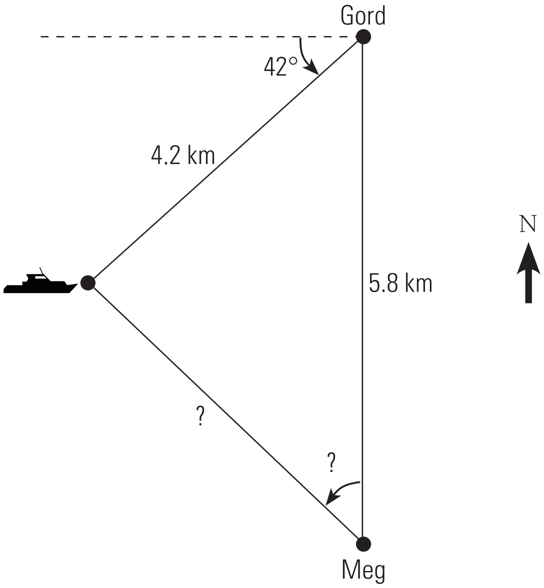


4. Truong is building a treehouse for his children, and he wants it to be shaped as a triangular prism, like a flatiron building. He plans the walls to be 1.8 m, 2.2 m, and 2.4 m long. At what angles will the walls meet?

Flatiron buildings are in the shape of triangular prisms, and are so named because they look like the irons that were once used to iron clothes before electricity. This flatiron building is the Hotel Europe in Vancouver, BC. It was built in 1909, and is currently used as affordable housing.

A flare gun kit is essential in every boat. Flare guns must be shot directly above so that the signal is visible for a longer period of time and allows for a more accurate location of the person in distress.

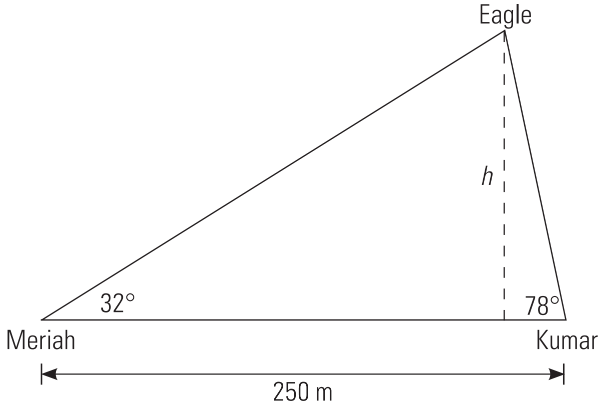
5. A boater sends an SOS call indicating that he is stranded on a point of land. Gord receives the call and determines that the boat is 3.2 km away from him at a direction of 42° south of west. Meg is at a station 5.8 km south of Gord.



a) How far is Meg from the stranded boater?

b) In what degree west of north would Meg have to travel to get to the stranded boater?

6. Meriah sees an eagle at an angle of elevation of 32° at the same time as Kumar spots it at an angle of elevation of 78°. They are on level ground, 250 m apart and on opposite sides of the eagle.



a) How far are Meriah and Kumar each from the bird?

b) At what height is the eagle flying?

The bald eagle is the national bird of the United States and a culturally significant animal to some Native Americans. In Canada, the majority of bald eagles breed in British Columbia.

Chapter 8

Owning a Small Business

A food vender sells Japanese delicacies such as mochi (soft rice cakes stuffed with sweet fillings) and almond cookies.

8.1 Start a Small Business

REVIEW: WORKING WITH COMPOUND INTEREST

compound interest: interest calculated on the principal plus interest accumulated in prior compounding periods

principal: the original amount of money borrowed

In this chapter, you will calculate the costs of operating a business. Businesses often need to take out loans to cover costs. To calculate the total cost of a loan, you need to calculate **compound interest**. Compound interest is calculated on the **principal**, plus any interest previously earned. For example, if you borrow money for two years but interest is compounded annually, the second year of interest will be calculated on the principal plus one year of interest.

The formula for compound interest is:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A is the amount of money you have to repay for the loan.

P is the principal (the amount you borrowed).

r is the annual interest rate as a decimal.

n is the number of **compounding periods** per year.

t is the **term** of the loan in years.

compounding periods: the time between calculations of interest

term: the length of time over which money is borrowed

Example 1

Michel takes out a loan for $2000.00, borrowed at an interest rate of 3.00% per annum, compounded monthly for 2 years.

Per annum means per year.

a) How much will Michel repay in total for the loan?

b) What will his monthly payments be?

SOLUTION

a) First, change the interest rate to a decimal: 3.00% is equal to 0.03. The principal is $2000.00 and the term is 2 years. The compounding period is monthly, so there are 12 compounding periods per year.

Use the formula for compound interest.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = (\$2000.00)\left(1 + \frac{0.03}{12}\right)^{12\times 2}$$

$$A = (\$2000.00)(1 + 0.0025)^{24}$$

$$A = (\$2000.00)(1.0025)^{24}$$

$$A \approx \$2123.51$$

Michel will pay back a total of $2123.51 on the loan.

b) Calculate how many monthly payments Michel will make over the 2-year term.

12 payments/year × 2 years = 24 payments

Each payment will be equal, so divide the total amount due by 24.

$2123.51 ÷ 24 ≈ $88.87

Michel's monthly payments on the loan will be $88.48.

BUILD YOUR SKILLS

The Bank of Montreal is the oldest and fourth largest bank in Canada. Some of its branches are located in buildings that are listed on the Registry of Historical Places in Canada. The building shown was constructed in 1894 in Montreal.

1. Calculate the total amount to be repaid and the monthly payments for the following loans.

   a) \$8000.00 borrowed at 4.50% per annum, compounded semi-annually for a 2-year term.

   b) \$8000.00 borrowed at 4.50% per annum, compounded semi-annually for a 5-year term.

2. Graham needs a \$5000.00 loan. One bank offers him a rate of 3.00% per annum, compounded quarterly for 2 years. Another bank offers him a rate of 2.25% per annum, compounded monthly for 3 years.

   a) How much would Graham have to repay for each loan?

   b) What would be the monthly payment in each case?

   c) Give one reason why Graham might choose each of the banks' loans.

NEW SKILLS: CONSIDERING SMALL BUSINESS OPPORTUNITIES

Running a small business is often time-consuming and challenging, but the results can be rewarding if you plan well. Business owners usually start with an idea of something they are interested in, be it a service business or a retail business.

Before opening a business, you must do some research to determine:

- Is the business right for you? Are you committed to it and will you stick with it even over rough periods?
- Is the business feasible? That is, is it likely to be financially profitable?
- Who are your potential customers or clients? Are there enough potential customers to support your business?

Market research will help you to answer these questions.

For more details, see page 276 of *MathWorks 12*.

Example 2

Rayyan is an early childhood educator, and she is thinking of opening her own daycare. She currently lives between Moose Jaw and Regina, SK, and is trying to decide where to open her daycare.

Opening a daycare for children with special needs may require special toys, equipment, and training.

Moose Jaw has a total population of 33 274, approximately 6% of which is 5 years old and under. There are 30 licensed daycares operating in the city.

Regina has a total population of 193 100, approximately 6% of which is 5 years old and under. There are 142 licensed daycares operating in the city.

a) Approximately how many children aged 5 and under are there in Moose Jaw and in Regina?

b) How many children aged 5 and under are there in each city for each daycare?

c) In which location would you recommend Rayyan open her daycare?

SOLUTION

a) Calculate 6% of each of the populations.

Moose Jaw: $33\ 274 \times 0.06 \approx 1996$
Regina: $193\ 100 \times 0.06 \approx 11\ 586$

There are about 1996 children aged 5 and under in Moose Jaw, and about 11 586 in Regina.

b) Divide the population aged 5 and under by the number of daycares.

Moose Jaw: $1996 \div 30 \approx 67$
Regina: $11\ 586 \div 142 \approx 82$

There are about 67 children aged 5 and under in Moose Jaw per daycare, while in Regina there are about 82 children for each daycare.

c) Even though there are more daycares already operating in Regina to compete with, there are more children for each daycare in Regina than in Moose Jaw. Therefore, Rayyan may find more potential clients in Regina.

BUILD YOUR SKILLS

3. Based on demographics, which city do you think would be a better location to start a fitness class for youth under the age of 15? Why?

POPULATION BY AGE

Age	*Courtenay, BC*	*Duncan, BC*
0–14	16%	17%
15–64	65%	65%
65–79	14%	13%
80+	5%	5%
Total population	49 215	41 385

Stretching after exercise helps increase one's flexibility and range of motion.

4. Dominic is planning to run computer classes for seniors at a community centre. The town's population is 29 063. About 21% of the population is over the age of 64.

    a) How many potential students are there for Dominic's classes?

    b) If only 3% of the seniors are interested in computer classes, how many students can Dominic expect?

NEW SKILLS: WORKING WITH START-UP AND OPERATING COSTS

When you start a small business, you have to take into account all of the **start-up costs**. That is, you have to calculate what amount you will have to invest or spend before you can open for business. If you do not have enough money saved, you may have to take out a loan or adjust your spending.

start-up costs: the goods and services you may need to purchase before you begin operating your business

When considering starting a business, you must be careful to think about all the items that will be needed, including the cost of a business licence. You should also allow for some unexpected costs.

Start-up costs are often one-time costs, such as buying a cash register for a store. In addition to these expenses, you will also need to consider **operating expenses**. These are expenses that are paid to keep the business running, and are often recurring, such as paying a monthly phone bill.

operating expenses: the ongoing cost of space, equipment, supplies, and other items required to run a business

For more details, see page 280 of *MathWorks 12*.

Example 3

Hairdressers require a licence and training to practise their craft.

Hazel completed a hairdressing program five years ago at the College of the Rockies in Cranbrook, BC, and has been working in a salon ever since. She has decided that she would like to open her own salon. She already has many of the items that she will need because she had to supply her own scissors and razors, curling irons and straighteners, brushes, perm curlers, and combs for her current job. List other items she is likely to need to start her own salon.

SOLUTION

Hazel will still need many other items. Some of the basics will include:

- a venue from which to work
- professional styling chairs
- hair washing sink and chair
- desk, mirrors, and cabinets
- cash register
- business licence
- telephone, computer
- laundry facilities
- professional hair dryers
- hair dyes and foils
- towels and capes
- advertisement

BUILD YOUR SKILLS

5. Dean is opening a dog daycare in his community. He has researched the market and has decided that the business is feasible. List at least 5 items that Dean needs to consider as start-up costs.

6. Sumalee is starting a home repair business, focusing on helping elderly people who cannot do minor repairs themselves. List at least 5 items she must consider as start-up costs.

National Seniors Day in Canada is on October 1. Honour this day by helping a senior family member, friend, or neighbour with a household chore.

Example 4

In the above example, you listed start-up costs for Hazel's hair salon. Hazel has considered the items, and has calculated the following costs:

Item	*Cost*
First month's rent	\$1000.00
Two professional styling chairs	\$300.00 each
Two hair washing sinks and chairs	\$700.00/set
Desk, mirrors, and cabinets	\$1000.00
Towels and capes	\$600.00
Washer and dryer (used)	\$1000.00
Cash register	\$75.00
Business licence	\$200.00/year

a) What are her total start-up costs?

b) If Hazel has \$5000.00 saved, does she have enough money to cover her expenses? If not, Hazel does not want to take out a loan. Suggest two ways she could adjust her start-up costs.

SOLUTION

a) Calculate Hazel's total start-up costs.

\$1000.00 + 2(\$300.00) + 2(\$700.00) + \$1000.00 + \$600.00 + \$1000.00 + \$75.00 + \$200.00 = \$5875.00

Hazel's total start-up costs are \$5875.00.

b) Hazel does not have enough money to cover her start-up costs.

$5875.00 – $5000.00 = $875.00

She has a shortfall of $875.00.

Hazel might do one of the following to adjust her start-up costs:

- She could install just one hair washing sink and chair.
- She could consider renting a chair in an existing salon, rather than opening a new location.
- She could take the laundry to a laundromat, instead of installing a washer and dryer.

BUILD YOUR SKILLS

A van equipped for professional carpet cleaning. Professional carpet cleaners use heavy-duty equipment that can vacuum dirt and steam-clean stains at the same time.

7. Marcus is beginning a home-based housecleaning business. He has $2000.00 saved up, and estimates his start-up costs as follows.

Item	*Cost*
Vacuum	$850.00
Brooms and mops	$175.00
Cleaning solution	$100.00
Buckets and other supplies	$75.00
Business licence	$125.00
Printing signs for his business	$50.00

a) Does he have enough money saved to start his business? What is the difference between his cash on hand and his start-up costs?

b) Suggest two additional items that Marcus should consider as start-up expenses for his business.

8. Aurora plans to open a daycare in her home. She estimates her start-up costs to be $4000.00 to do remodelling and get a permit. She has $1500.00 saved, and can save $350.00/month in her present job.

   a) How long will it be before she has saved enough money so she can open her daycare?

   b) Suggest two additional costs that Aurora should consider when starting her business.

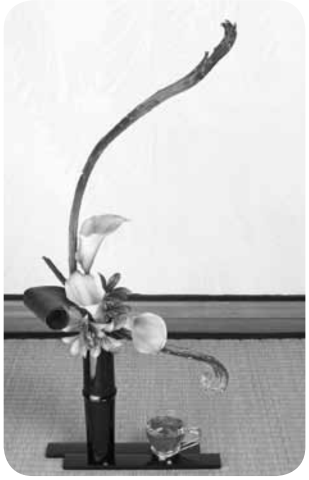
9. Ari plans on starting a trash removal business. He has a three-quarter ton truck that he plans to use, but estimates that he will have the following start-up costs.

Item	*Cost*
Tools	$300.00
Business licence	$175.00
Website development cost	$150.00

   a) Ari does not have any money saved up, so he needs to take out a loan from the bank. How much money will he have to borrow?

   b) The loan has an interest rate of 5.05% per annum, compounded monthly, for a 1-year term. How much will Ari have to repay?

Example 5



In Japan, the art of flower arrangement is called *ikebana*, which translates as "giving life to flowers." *Ikebana* is a disciplined art form that is typically minimalist in style and is based on a scalene triangle that symbolizes heaven, earth, and humanity or sun, moon, and earth.

Irina has worked for several years in a flower shop. The current owner has decided to retire, and Irina is considering buying the business. Since the business is already operating and successful, the start-up costs are minimal other than the purchase price. However, Irina has to consider the business's monthly operating expenses. What expenses will she have to consider?

SOLUTION

Irina has to consider many operating expenses to keep the flower shop running. Some such items include:

- the flowers
- other floral supplies, such as ribbon, vases, flower pots, and so on
- mortgage payments
- utilities
- telephone and internet bills
- employees' wages
- bank account fees
- advertising costs (including website maintenance)
- delivery costs (including cost of vehicle maintenance, gas, insurance)
- insurance
- annual business licence
- property taxes

BUILD YOUR SKILLS

10. Zac enjoys creating silver jewellery. He has sold many pieces at craft fairs and to friends. He has been encouraged to open a small retail booth in a mall. After paying his start-up costs and obtaining a business licence and a venue, what operating expenses must he consider? Suggest at least 5 expenses.

11. Jarrod has saved up $1500.00 to open a house painting business that he will run out of his home. He already has a truck to use for the business. He has listed the following expenses.

 a) Categorize the expenses as start-up costs or operating costs.

Item	*Cost*	*Start-up or Operating?*
Business licence	$175.00/year (paid in full at start-up)	
Gasoline	$175.00/month	
Vehicle insurance	$200.00/month	
Ladders	$650.00	
Coveralls and clothes	$175.00	
Cleaning cloths	$25.00	
Canvas sheeting	$285.00	
Paint brushes and trays	$150.00	
Phone bill	$50.00/month	
Trowels, crack filler, sand paper	$75.00	
Miscellaneous	$100.00	

 b) How much money does Jarrod need to start his business and operate it for the first month?

Gelato is the Italian word for ice cream. The difference between gelato and ice cream is in the way it is made: gelato is frozen quickly in smaller, individual batches whereas ice cream is frozen with a continuous assembly line freezer.

12. Ella is starting a small business operating an ice cream cart at beaches and parks in her community. She already owns a truck that she will use to transport her cart around the town. She has made the following list of expenses.

a) Categorize her expenses as start-up costs or operating costs.

Item	*Cost*	*Start-up or Operating?*
Ice cream	\$500.00/month	
Ice cream cart	\$3000.00	
Business licence	\$200.00/year (paid in full at start-up)	
Insurance	\$2500.00/year (paid in full at start-up)	
Printing a sign for her cart	\$100.00	
Gasoline	\$200.00/month	

b) How much will it cost Ella to start up her business and operate it for the first month?

c) What percentage of her expenses are one-time costs?

d) Ella has \$1000.00 saved up to use towards starting the business. How much will she have to borrow from the bank? If she gets a loan at 4.50% per annum, compounded quarterly, for a term of 2 years, how much will she have to repay?

PRACTISE YOUR NEW SKILLS

1. Anna noticed that many shoppers at the mall like to sit and rest for a few minutes on the benches. She came up with the idea to open a foot massage booth for them. The mall management has told her there are two possible locations for her booth. Anna decides to observe the foot traffic in the two locations.

   The first location is outside The Toy Top, a toy store where an average of 86 people pass per hour, usually mothers with children. The second is outside Handsome Home, a home decor store, where 35 people pass per hour, mostly adults alone or in groups of two.

   Which do you think is the best location for Anna's foot massage booth and why?

2. Fabian is starting up a tour company that will take people on bicycle tours around his city. List at least 5 start-up costs that he needs to consider.

City bicycle tours are a unique way to discover a city. In New Orleans, the Lower Ninth Ward Rebirth Bike Tour takes people through the Lower Ninth Ward, the area most devastated by Hurricane Katrina in 2005, to meet locals and see the ongoing rebuilding efforts of the community. Partial proceeds from the tour go to local charities.

3. Rena is starting a small business making jams and jellies, which she will sell at local markets. Categorize the following expenses as start-up or operating costs.

JAM AND JELLY BUSINESS START-UP AND OPERATING COSTS	
Expense	*Start-up or Operating?*
Business licence	
Cash box	
Signs for booth	
Jars	
Fruit and other ingredients	
Booth cost (per market date)	

4. Michael is starting a window-washing business.

Window cleaners who specialize in large or tall buildings usually require licences and extra insurance coverage.

a) He has estimated that he will have the following costs. Categorize the expenses as start-up or operating costs.

Item	*Cost*	*Start -up or Operating?*
Ladder	$250.00	
Other tools	$500.00	
Business licence (annual)	$125.00	
Advertising signs	$50.00	
Truck (used)	$15 000.00	
Signage for truck	$100.00	
Phone bill (monthly)	$80.00	
Vehicle insurance (monthly)	$200.00	
Gasoline (weekly)	$50.00	
Vehicle maintenance (annual)	$500.00	

b) How much money will Michael need to start his business?

c) Michael has \$5000.00 saved to start his business. In order to cover the rest of his costs, he will take out a loan. The bank offers him a loan with an interest rate of 6.50% per annum, compounded quarterly, for a 4-year term. How much will he repay on the loan?

5. Zhou needs a \$4000.00 loan to start his business. He has been offered the following loans by two different banks.

   Option 1: interest rate of 4.50% per annum, compounded annually, for a 3-year term

   Option 2: interest rate of 5.50% per annum, compounded monthly, for a 2-year term

   a) How much would Zhou need to repay for each option?

   b) What would be the monthly payments for each option?

   c) Which option should Zhou choose? Explain your reasoning.

8.2 Operate a Small Business

NEW SKILLS: WORKING WITH REVENUE AND EXPENSES

revenue: amount earned through the sale of goods or services

fixed expenses: expenses that occur on a regular basis and do not change in value

variable expenses: expenses that change in value or frequency

In the previous section, you looked at the start-up costs and operating costs of various businesses. In order to run a business, you also have to consider **revenue** (income) and compare it to your expenses.

For your expenses, you also need to determine whether they are **fixed** or **variable**. A fixed expense is a regularly recurring payment that is always the same amount, such as monthly rent. A variable expense changes in value or frequency, such as a payment for an emergency repair.

For more details, see pages 287–288 of *MathWorks 12*.

Example 1

Rafaela runs a catering service preparing and delivering frozen dinners to the elderly. During the past month she listed the following items in her financial records.

a) Indicate which items are revenue and which are expenses. Calculate the total of each category.

b) Classify each expense as fixed or variable.

Groceries	$875.23	Cheque from Mr. Rogers	$309.91
Payment from Mrs. Harrison	$476.43	Refund to Mr. Zibit	$25.34
Gas for car	$296.74	Payment from Mr. Oswich	$421.39
Payment from Mr. Dorn	$403.43	Employee salary	$1254.25
Food containers	$72.84	Payment from Mrs. Zedrickowsky	$376.76
Payment from Mrs. Towers	$483.72	Monthly fee paid for advertisement	$100.00
Tip from Mrs. Towers	$20.00	Telephone bill (flat rate plan)	$80.00
Groceries	$141.51	Payment from Mr. Ozaki	$395.62
Car insurance	$106.53	Payment from Ms. Li	$508.26

SOLUTION

Pizza is not the only kind of food being delivered anymore. Some companies offer food delivery services for organic, locally grown produce.

a) Money that Rafaela receives is classified as revenue, and money that she pays is an expense.

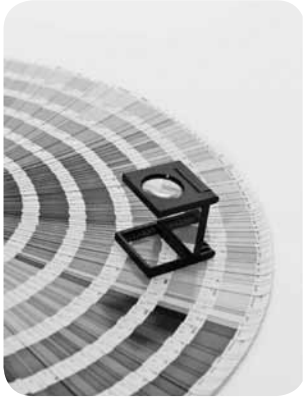
Revenue		*Expenses*	
Payment from Mrs. Harrison	$476.43	Groceries	$875.23
Payment from Mr. Dorn	$403.43	Gas for car	$296.74
Payment from Mrs. Towers	$483.72	Food containers	$72.84
Tip from Mrs. Towers	$20.00	Groceries	$141.51
Cheque from Mr. Rogers	$309.91	Car Insurance	$106.53
Payment from Mr. Oswich	$421.39	Refund to Mr. Zibit	$25.34
Payment from Mrs. Zedrickowsky	$376.76	Employee salary	$1254.25
Payment from Mr. Ozaki	$395.62	Monthly fee paid for advertisement	$100.00
Payment from Ms. Li	$508.26	Telephone bill (flat rate plan)	$80.00
Total revenue	$3395.52	**Total expenses**	$2952.44

Rafaela's total revenue was $3395.52 and her total expenses were $2952.44 for the month.

b) Fixed items are the same from month to month, and variable items change in value or frequency.

EXPENSES	
Fixed	*Variable*
Car Insurance	Groceries
Employee salary	Gas for car
Monthly fee paid for advertisement	Food containers
Telephone bill (flat rate plan)	Refund to Mr. Zibit

BUILD YOUR SKILLS



A useful tool for most graphic designers is a colour guide book. A designer can use the guide book to "colour match" during the design stage. The colour chosen from the guide should match the final printed product.

1. Sam runs a graphic design business with one employee. His revenue and expenses for one month are shown below. Classify each item as an expense or revenue, and as fixed or variable.

Item	*Revenue or Expense?*	*Fixed or Variable?*
Rent		
Business insurance		
Employee wage		
Telephone and internet bill (flat rate plan)		
Tip from client		
Utility bill		
Bill for printer cartridges from supplier		
Payment for design services		
Monthly website maintenance fee		
Bill for paper from supplier		

2. Kate and Marc operate a carpet cleaning business. They own a van in which they can transport their equipment. They have identified the following expenses.

   a) Classify each expense as fixed or variable.

Item	*Cost*	*Fixed or Variable?*
Carpet shampooer repair	$175.00	
Shampoo	$125.00	
Advertising (ongoing ads in newspaper)	$75.00/month	
Business licence	$110.00/year	
Van insurance	$125.00/month	
Gas	$300.00/month	
Salaries for Kate and Marc	$2160.00/month each	
Telephone (fixed rate plan)	$70.00/month	

   b) What would their total fixed expenses be for one month of operation? Assume they have already paid their annual fees.

3. Edwin runs a hot dog stand at the beach during the summer. In one month, he had the following revenue and expenses.

   a) Identify each item as either an expense or revenue, and fixed or variable.

Item	*Cost*	*Expense or Revenue?*	*Fixed or Variable?*
Loan payment for purchase of stand	$175.00		
Monthly business licence	$45.00		
Food supplies	$1700.00		
Wages for employee	$525.00		
Insurance	$175.00		
Hot dog sales	$3500.00		
Tips	$92.00		

   b) Calculate his total expenses and revenue.

NEW SKILLS: WORKING WITH PROFIT, LOSS, AND THE BREAK-EVEN POINT

The goal of a business is usually to earn money. To determine if your business is earning money, you have to compare your revenue to your expenses. If you earn more than you spend, your business will have a **profit**. If you spend more than you earn, your business will have a **loss**. If your revenue and expenses are the same, your business will **break even**.

For more details, see page 287 of *MathWorks 12*.

profit: the amount by which revenue exceeds expenses for a given period

loss: the amount by which expenses exceed sales for a given period

break even: the point at which business expenses equal sales revenue

Traditional French macaroons consist of a buttercream filling sandwiched between two meringue-based cookies. They come in a variety of flavours and colours including green tea matcha, raspberry, and chocolate.

Example 2

Vaughn runs a bakery, making baked goods that he sells to cafés. He owns the van that he uses for purchases and delivery. His revenue and expenses for one month are as follows.

Bakery monthly revenue and expenses	
Rent	$1200.00
Wages	$3200.00
Sales	$10 000.00
Loan payment	$226.00
Food and ingredients	$3000.00
Utilities	$150.00
Insurance	$250.00
Van insurance	$195.00
Gas for van	$225.00
Salary for Vaughn	$2200.00

a) Calculate Vaughn's profit or loss for the month.

b) What would Vaughn's sales have to be for the month to break even?

c) If Vaughn cannot increase his sales, suggest a way for him to break even.

SOLUTION

a) Classify each item as revenue or expense, and calculate the total for each category.

BAKERY MONTHLY REVENUE AND EXPENSES			
Revenue		*Expenses*	
Sales	$10 000.00	Rent	$1200.00
		Wages	$3200.00
		Loan payment	$226.00
		Food and ingredients	$3000.00
		Utilities	$150.00
		Insurance	$250.00
		Van insurance	$195.00
		Gas for van	$225.00
		Salary for Vaughn	$2000.00
Total revenue	$10 000.00	**Total expenses**	$10 646.00

Vaughn's expenses are higher than his revenue, so he has a loss.

$10 646.00 – $10 000.00 = $646.00

Vaughn has a loss of $646.00 for the month.

b) In order to break even, Vaughn's expenses must equal his revenue. He would have to make sales of $10 646.00.

c) If Vaughn cannot increase his sales, he will have to decrease his expenses in order to break even. He could consider doing one of the following:

- reducing the hours of one or more of his employees, to reduce the amount he has to pay in wages;
- buy less expensive food and ingredients; or
- decrease his own salary.

BUILD YOUR SKILLS

4. Serge runs a delivery service, delivering large items such as furniture and office equipment for various companies. He has three trucks and drivers working for him. He charges a flat rate of $75.00 per delivery. His expenses for one month are as follows.

Expenses	
Office supplies	$75.00
Fuel costs	$965.00
Cell phones	$225.00
Insurance on trucks	$685.00
Salaries for drivers	$8225.00
Advertising	$50.00
Truck maintenance	$600.00
Salary for Serge	$5000.00

a) During the month, one truck made 75 deliveries, the second made 45 deliveries, and the third made 92 deliveries. Did the company have a profit or a loss?

b) How many deliveries would the company have to make to break even?

In silk-screen printing, ink is rolled along a woven mesh stencil made of silk. The ink is forced through the mesh and onto a printing surface such as a T-shirt.

5. Linnea plans to open a silk-screening shop where she will print custom designs onto T-shirts. She needs to purchase a printer and ink, which she estimates will cost about $2750.00. If she can buy T-shirts in bulk for $5.85 each, and she plans on selling the shirts for $14.95, how many shirts will she have to sell before she covers the cost of the printer?

6. Chanh plans to offer a motorboat service for tourists in his hometown of Echo Bay on Saturna Island, BC, for four months of the year.

   - He bought a used boat for $18 250.00, which he paid for with a bank loan at an interest rate of 4.50% per annum, compounded quarterly over 6 years. He has to make the monthly payments on the loan.
   - His business licence is $25.00/month.
   - His insurance is $212.00/month.
   - Fuel will cost $225.00/month.
   - He estimates his other business expenses to be $200.00/month.
   - Chanh needs to pay himself a salary of $3000.00/month to cover his living expenses.

   a) What are Chanh's total expenses for one month?

b) If Chanh offers 2-hour rides for $90.00/ride, how many rides must he provide in one month to break even? Does this seem reasonable? Explain your answer.

Saturna Island is part of the Gulf Islands chain in British Columbia. The name "Saturna" comes from the name of the Spanish naval schooner—*Santa Saturnia*— that was used by Spanish explorers when they sailed the area in 1791.

NEW SKILLS: MAKING PROFITABLE BUSINESS DECISIONS

When you own a business, you must make decisions that affect its success and profitability. For example, if you run a retail business, you have to set prices high enough to cover the cost of materials and your other expenses, but not so high that you turn customers away.

A number of factors can affect the success of your business, including:

- the location of the business
- where you buy your goods and materials
- how much you charge for your goods or services
- the hours or times of the year of operation
- the number of employees
- how you advertise

Example 3

Parker operates a home renovation business. He has estimated that the cost of materials for a client's renovation will be $5685.00, and that the job will take 18 hours of labour to complete.

a) Parker is deciding whether to charge the client the cost of the materials plus $35.50 per hour, or a flat rate of 115% of the cost of materials. Which option would give Parker a higher income?

b) Assume Parker charges the client the higher price. He hires two employees to complete the renovation in less time. They can complete the job in 9 hours, but Parker will have to pay each employee $18.00/hour. What is Parker's profit after paying the employees?

c) Suggest two changes that Parker could make to earn a higher profit.

SOLUTION

a) Calculate Parker's income if he charged \$35.50/hour plus the cost of materials.

earnings = \$5685.00 + (18 h × \$35.50/h)
earnings = \$5685.00 + \$639.00
earnings = \$6324.00

Calculate Parker's income if he charged 115% of the materials cost.

earnings = 1.15 × \$5685.00
earnings = \$6537.75

Charging 115% of the cost of materials would give Parker a higher income..

b) Calculate Parker's profit.

\$6537.75 – \$5685.00 = \$852.75

Calculate how much Parker would have to pay his employees.

2 × \$18.00/h × 9 h = \$324.00

Calculate Parker's profit after paying the employees.

\$852.75 – \$324.00 = \$528.75

Parker would earn a profit of \$528.75.

c) To increase his profit, Parker could make some of the following changes:

- He could hire just one employee, instead of two, and do more work himself.
- He could find a cheaper source for the materials.
- He could increase his flat rate to 120% or more.

BUILD YOUR SKILLS

7. Rajveer runs a lemonade stand at Crescent Beach in White Rock, BC. He runs the stand from 8 am to 10 pm, 7 days a week, year-round. He employs 4 staff members. He is not breaking even, and needs to make some adjustments to the business to increase its profitability. Suggest 4 changes Rajveer could make to increase the profits of his lemonade stand.

Crescent Beach is a popular place for swimming, flying kites, skim boarding, crab fishing, and clam digging.

8. Zoe can rent space for her gelato stand for $275.50 per month for the year, or for $575.25/month for only the four summer months. She is debating whether she should run the business all year, or only during the summer months. She knows that after all other expenses, not considering rent, she makes a profit of $700.00/month for each of the summer months, but only $150.00 for each of the rest of the months. She is paying herself a wage in her expenses.

   a) What is Zoe's profit or loss if she operates for the entire year?

   b) What is Zoe's profit or loss if she operates for only the four summer months?

   c) What would you recommend Zoe do: operate the stand all year, or only during the summer? Explain your reasoning.

PRACTISE YOUR NEW SKILLS

1. Ephraim runs a bookkeeping business. His revenue and expenses for one month are shown below. Classify each item as revenue or expense, and fixed or variable.

Item	*Revenue or Expense?*	*Fixed or Variable?*
Rent		
Insurance		
Office supplies (printer paper, ink)		
Monthly payment from regular client		
Utility bill		
Telephone and internet bill (flat rate plan)		
Payment from client for a one-time job		
Repair bill for fixing printer		

2. Zophia has started a housecleaning business. She has hired 4 employees. Zophia pays each worker $18.00/hour and provides them with all cleaning supplies. She charges clients $50.00/hour to have two employees clean their houses. Below is a list of Zophia's revenue and expenses for a month.

Some household cleaning products can be toxic. Some non-toxic, all natural, yet effective cleaning products are white vinegar, lemon juice, and baking soda.

a) Classify each item as an expense or revenue.

Item	*Value*	*Revenue or Expense?*
Cheques from customers	$700.00	
Cleaning supplies	$24.86	
Cheques from customers	$500.00	
Employee wages	$568.00	
Employee wages	$496.00	
Cash from customers	$550.00	
Cleaning supplies	$65.32	
Cheques from customers	$600.00	
Cash from customers	$300.00	
Employee wages	$512.00	
Cheques from customers	$250.00	
Employee wages	$512.00	
Vacuum repair	$64.24	

b) Did Zophia's business have a profit or a loss for the month? Calculate the profit or loss.

c) Suggest two ways Zophia could increase her profits.

3. Pippa has decided to publish a book of her photographs of flowers. She is considering two publication options.

   Nature Publishers has offered her a 10% royalty—that is, 10% of the profit on the sales of the book. They would publish 5000 books, which they would sell for $35.95 each. Publishing costs would be $6000.00.

   Pippa has also considered self-publishing, which would cost $150 000.00 if she were to publish 5000 books. She would sell the books for $35.95.

   a) Calculate Pippa's possible profit for each option.

   b) Which option would you suggest Pippa choose? Why?

The floral emblem of the Yukon Territory is fireweed. The name "fireweed" comes from the tendency for the plant to grow in areas burned down by forest fires.

4. Lenny and his friends have a band. They have created a logo for their band and have had it printed on T-shirts to sell at their gigs. Each T-shirt cost them $12.49 to make, and each of the 5 members of the band and 6 crew got a free T-shirt.

   If they had 500 shirts printed, how much would they have to sell each shirt for, after giving away the free shirts, if they wanted to earn a profit of $2000.00 on the sale of the shirts?

5. Marcel is an electrician. He charges $45.00 per house call, plus $26.50/hour. He has found that his clients do not like the $45.00 flat fee, so he wants to incorporate the fee into his hourly rate. He knows that house calls take an average of 3 hours.

   a) Using his current fee, how much does Marcel charge for the average 3-hour house call?

   b) How much would he have to charge per hour if he wanted to eliminate the $45.00 flat fee but still earn the same amount per 3-hour house call?

   c) After changing his rate, Marcel found he was not making as much of a profit as with his previous fee. Suggest one possible reason for his lower profit.

Buy or Lease a Vehicle for Your Business 8.3

NEW SKILLS: BUYING AND LEASING VEHICLES

Many businesses need a vehicle in order to operate. If you need a car, truck, or other vehicle for your business, you have a few different payment options:

- You can buy the vehicle by paying the full purchase price including tax.
- You can pay a down payment and take out a loan for the rest of the purchase price. You will then have to make monthly payments on the loan.
- You can **lease** the vehicle, making regular monthly payments over a fixed term.

lease: an agreement to pay for the use of a vehicle for a specific period of time; similar to renting

If you lease a vehicle, you will have to sign an agreement that may include the following conditions:

- lease term: the length of time you will be using the vehicle;
- lease rate: the amount of your monthly payments (taxes and interest charges are usually built into the monthly payments);
- security deposit: a payment made at the start of the lease term, which is refunded if the vehicle is returned in good condition at the end of the term;
- kilometre allowance: the maximum distance that can be driven over the term of the lease;
- delivery charge: a payment made at the start of the lease term, to cover environmental fees and taxes; and
- option to purchase: you may be able to buy the vehicle at the end of your lease term, based on its **residual value**.

residual value: the vehicle's value when the lease is over, usually specified in the lease agreement

When deciding between buying and leasing, you need to consider the costs of each option, how you plan to use the vehicle, and how long you plan to keep it.

For more details, see page 298 of *MathWorks 12*.

Example 1

In North America, pickup trucks are classified in four sizes: half ton, three-quarter ton, one ton, and one and a half ton. The size ranges were derived from the maximum load of the vehicle, but nowadays pickup trucks can carry more than their classification.

Roger, who lives in Manitoba, plans on purchasing a three-quarter ton truck. The purchase price is $29 499.00. He must also pay 5% GST and 7% PST.

a) What is the total price of the truck?

b) Roger does not have enough money saved to pay the full purchase price. He can make a down payment of $10 000.00, then take out a bank loan for the rest. The bank offers him a loan at an interest rate of 4.50% per annum, compounded annually, for a 4-year term. What will Roger pay in total for the truck?

SOLUTION

a) Calculate the GST and PST payments.

$29 499.00 × 0.05 = $1474.95
$29 499.00 × 0.07 = $2064.93

Calculate the total purchase price.

$29 499.00 + $1474.95 + $2064.93 = $33 038.88

The total price of the truck is $33 038.88.

b) Calculate how much money Roger will need to borrow from the bank.

$33 038.88 – $10 000.00 = $23 038.88

Use the compound interest formula to calculate how much Roger will repay on the loan.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = \$23\,038.88\left(1 + \frac{0.045}{1}\right)^{1\times 4}$$

$$A = (\$23\,038.88)(1.045)^{4}$$

$$A = \$27\,474.29$$

Calculate the total cost of the truck.

$10 000.00 + $27 474.29 = $37 474.29

If Roger takes out a loan, he will pay $37 474.29 for the truck.

BUILD YOUR SKILLS

The first pickup truck was based on the Ford Model T car and sold for US $281.00 in 1925.

1. Stefania lives in Broderick, SK, and plans to buy a half-ton truck. She has two options:

   - She can purchase a new truck, with only basic features, for $22 789.00 plus 5% GST and 5% PST.
   - She can buy a used one with approximately 120 000 km on it, but with many upgraded features, for the same price plus 5% GST (PST is not paid on used vehicles in Saskatchewan).

   a) Calculate how much Stefania would have to pay for each truck.

   b) Give one reason in support for and one reason against Stefania purchasing each truck.

Some courier companies employ bicycle messengers or cycle couriers to deliver items within the central business district of major cities, because cyclists can manoeuvre through busy traffic much more easily than cars.

2. Gurvir needs to buy a small car for his courier business in Edmonton, AB. He has two options:

   - He can buy the car for \$19 195.00 plus 5% GST.
   - He cannot pay in full so he must make a down payment of \$5000.00, then take out a loan to pay the rest of the purchase price. The bank offers him a loan at an interest rate of 2.50% per annum, compounded monthly for 3 years.

   a) What is the total purchase price of the vehicle?

   b) If Gurvir makes the down payment and takes out the loan, how much more will he pay for the car than if he had paid the entire purchase price?

3. Gavin plans to buy 4 snowmobiles for his company in Yellowknife, NT. Each snowmobile is listed at \$9989.00, and he must pay 5% GST. Gavin has \$19 750.00 saved for a down payment. He takes out a loan for the rest of the purchase price at an interest rate of 3.20% per annum, compounded semi-annually for 4 years. What is the total cost of the snowmobiles?

Example 2

Francine needs a van for her catering company. She has found one that costs $25 675.00, including all fees and taxes. She has two options:

- She can lease the van for 4 years, for $450.00/month with a security deposit of $2000.00 and a delivery charge of $529.96.
- She can use the $2529.96 as a down payment and take out a loan at an interest rate of 3.15% per annum, compounded monthly for a period of 4 years.

a) Calculate the total cost of leasing the van for 4 years if Francine damages the van and doesn't get her security deposit back.

b) Calculate the total cost of purchasing the van, including the interest on the loan.

c) What would be the monthly payments on the loan?

d) Which option would you recommend Francine choose? Explain your reasoning.

SOLUTION

a) Calculate the number of monthly payments Francine will make.

$4 \times 12 = 48$ months

Calculate the total amount Francine would pay in monthly payments.

$450.00/month × 48 months = $21 600.00

Calculate the total cost of the lease.

total cost = total monthly payments + security deposit + delivery charge
total cost = $21 600.00 + $2000.00 + $529.96
total cost = $24 129.96

The total cost of leasing the van for 4 years is $24 129.96.

b) Calculate the amount Francine would have to borrow.

$25 675.00 – $2529.96 = $23 145.04

Use the compound interest formula to calculate the cost of the loan.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = (\$23\ 145.04)\left(1 + \frac{0.0315}{12}\right)^{12\times 4}$$

$$A = (\$23\ 145.04)(1.002\ 625)^{48}$$

$$A \approx \$26\ 248.67$$

Francine would repay $26 248.67 for the loan.

Calculate the total amount Francine would pay for the van.

total cost = down payment + amount repaid for loan
total cost = $2529.96 + $26 248.67
total cost = $28 778.63

Francine would have to pay $28 778.63 for the van.

c) To calculate the monthly payments on the loan, divide the amount to repay by the number of months of the loan term.

$26 248.67 ÷ 48 months = $546.85

The monthly payments on the loan would be $546.85.

d) Answers will vary. Although purchasing the van costs more in total and has higher monthly payments, she would own the vehicle at the end of 4 years. With the lease, she will have paid only a small amount less overall ($28 778.63 – $24 129.96 = $4648.67), but she will not own the vehicle after 4 years. However, the monthly payments for the lease are about $100.00 less; Francine may not be able to afford the higher monthly payments of the purchase option.

BUILD YOUR SKILLS

4. Elena will be leasing a delivery truck for her furniture business. The terms of the lease are as follows:

   - term: 5 years
   - security deposit: $1000.00
   - monthly payments: $529.95
   - delivery charge: $792.27

   a) How much will Elena have to pay on the day she first picks up the truck?

   b) Assuming Elena returns the truck in good condition at the end of the lease term, what is the total cost of the lease?

If starting a furniture delivery business, one way to get the business going is to call up local delivery stores and see if they offer delivery services. If they do not, try negotiating with them to start up a partnership.

5. Vern needs a truck for his business. He can choose one of the following options:

   - He can lease a truck for $687.00/month for 48 months. There is a $376.00 delivery fee, a $2000.00 down payment, and a $750.00 security deposit.
   - He can buy the truck by making a $2750.00 down payment, including taxes, and monthly payments of $850.00 for 48 months.

   a) Calculate the cost of leasing the truck for 4 years. Assume he will get the security deposit back at the end of the lease term.

b) Calculate the cost of buying the truck.

c) Which option would you recommend Vern choose? Why?

Certain breeds of dog may require their coat to be trimmed, cut, or stripped—the process of removing dead hair out of the coat of a non-shedding dog—for practical or cosmetic reasons.

6. Anne-Marie is leasing a vehicle for her pet grooming business. She will pay a \$4000.00 down payment, a \$1000.00 security deposit, and a delivery fee of \$425.00. The term of the lease is 36 months and the monthly payments are \$579.00.

a) How much must Anne-Marie pay at the beginning of the lease?

b) What will her total costs be if she loses half the security deposit due to damage done to the vehicle?

Example 3

Wen is planning to lease a vehicle to use for deliveries for his restaurant. The lease has the following conditions:

- term: 4 years
- monthly payments: $457.92
- security deposit: $500.00
- delivery charge: $629.05
- kilometre allowance: 25 000/year ($0.12/km for each additional kilometre)

The residual value at the end of the lease term is $13 880.00.

a) How much will it cost to lease the vehicle over a 4-year term? Assume Wen would return the vehicle in good condition, having driven 92 000 km.

b) If Wen purchases the vehicle at the end of the lease term, how much will he have paid for it? If he buys the vehicle at the end of his lease, his security deposit is returned and he is not required to pay if he goes over his kilometre allowance.

c) If Wen buys the vehicle instead of leasing it, it will cost $29 674.00, including taxes and fees. What is the difference in price between purchasing the vehicle and leasing it, then purchasing it at the end of the lease term?

SOLUTION

a) Calculate the number of monthly lease payments Wen will pay.

4 × 12 = 48 months

Calculate the total of the monthly lease payments.

$457.92 × 48 = $21 980.16

Calculate the total cost of the lease.

total cost = total monthly payments + delivery charge
total cost = $21 980.16 + $629.05
total cost = $22 609.21

The cost of leasing the vehicle for 4 years is $22 609.21.

b) Calculate the total cost of the vehicle by adding the lease cost to the residual value.

total cost to buy = $22 609.21 + $13 880.00
total cost to buy = $36 489.21

If Wen buys the vehicle at the end of the 4-year lease term, he will have paid $36 489.21 in total.

c) Calculate the price difference between buying outright and leasing, then buying at the end of the term.

$36 489.21 – $29 674.00 = $6815.21

Wen would pay $6815.21 more if he purchased the vehicle after leasing it than if he had purchased it outright.

BUILD YOUR SKILLS

Choosing the right vehicle to lease or purchase for your business takes careful consideration. Consider such things as a vehicle's size, engine power, fuel efficiency, resale value, and safety rating.

7. Candice needs a car for her sales business. The vehicle has a purchase price of $26 547.00 (including all taxes and fees). She has three options:

   - She can lease it at a cost of $465.00/month for 3 years and then purchase it at the end of the lease term for a residual cost of $13 539.00 cash.
   - She can lease it at a cost of $465.00/month for 3 years and then purchase it at the end of the lease term for a residual cost of $13 539.00, for which she takes out a loan at a rate of 3.00% per annum, compounded monthly for 1 year.
   - She can purchase it by taking out a bank loan at a rate of 2.50% per annum, compounded quarterly over a term of 3 years.

   What is the cost in each case?

8. The lease agreement for a car says that the residual value after 4 years is 45% of the purchase price of $32 789.54. What is the residual value?

9. Lainey is planning to lease a vehicle to use for her business. The lease has the following conditions:

- term: 5 years
- monthly payments: $295.00
- security deposit: $500.00
- delivery charge: $415.00
- kilometre allowance: 25 000/year ($0.15/km for each additional kilometre)

The residual value at the end of the lease term is $10 500.00 (including taxes and fees).

a) How much will it cost to lease the vehicle over a 5-year term? Assume Lainey returns the vehicle in good condition, having driven 130 000 km.

Hybrid vehicles use two or more power sources to move the vehicle. They are more fuel efficient and have lower emissions than regular vehicles.

b) If Lainey purchases the vehicle at the end of the lease term, how much will she have paid for it?

c) If she buys the vehicle instead of leasing it, it will cost $21 000.00, including taxes and fees. What is the difference in price between purchasing the vehicle and leasing it, then purchasing it at the end of the lease term?

PRACTISE YOUR NEW SKILLS

Some vehicles come equipped with in-dash global positioning systems as an added feature. It might be worth the extra cost if the vehicle will be used, for example, in a delivery business.

1. Mel wants to buy a car listed at $15 780.00. She wants to upgrade some of the features: a better stereo will cost an additional $879.00, better tires cost $476.00 more, and paint detailing will cost $539.00. Mel lives in Saskatchewan, where GST is 5% and PST is 5%. How much will Mel pay for the car?

2. Erik is buying a truck for his gardening business. He plans to make a down payment of $4200.00 then make monthly loan payments of $675.83 for a period of 48 months. What is the purchase price of the truck?

3. Zhijuan needs a car for her home business. She has chosen a model that costs $22 095.21, including all fees and taxes. She has two options:

   - She can lease the car for 4 years, for $356.15/month with a security deposit of $500.00 and a delivery charge of $450.00.
   - She can make a down payment of $1000.00 and take out a loan at an interest rate of 3.95% per annum, compounded quarterly for a period of 4 years.

   a) Calculate the total cost of leasing the car. Assume Zhijuan returns the car in good condition and receives her security deposit back.

   b) Calculate the cost of purchasing the car, including the interest on the loan.

   c) What would be the monthly payments on the loan?

   d) Which option would you recommend for Zhijuan? Explain your reasoning.

Off-road vehicles are capable of being driven on and off paved or gravel surfaces. They typically have flexible suspension and deeply treaded tires.

4. Marlon has just leased a sports utility vehicle for his tour guiding company. The conditions of the agreement are as follows:

   - term: 6 years
   - monthly payments: $345.00
   - security deposit: $1500.00
   - delivery charge: $500.00
   - kilometre allowance: 25 000/year ($0.11/km for each additional kilometre)

   The residual value at the end of the lease term is $16 350.00.

   a) How much will it cost to lease the vehicle over a 6-year term? Assume Marlon returns the vehicle in good condition, having driven 152 000 km.

   b) If Marlon purchases the vehicle at the end of the lease term, how much will he have paid for it in total?

   c) If Marlon buys the vehicle instead of leasing it, it will cost $32 984.73, including taxes and fees. What is the difference in price between purchasing the vehicle and leasing it, then purchasing it at the end of the lease term?

CHAPTER TEST

1. Explain what is meant by the following terms.

   a) start-up costs

   b) operating expenses

   c) profit

   d) break-even point

2. Classify the following items as start-up costs or operating expenses for a carpenter.

Item	*Start-up or Operating?*
Basic tools	
Ladder	
Rental storage space	
Truck payments	
Phone bill	
Gas	

3. Janelle is a massage therapist and would like to set up her own room in the existing health spa. She has estimated the following start-up costs.

Item	*Cost*
Rent	$680.00
Stools – 2	$70.00 each
Massage table	$416.00
Massage chair	$274.00
Blanket warmer	$89.00
Sound system	$415.00
Music – 5 albums	$15.00 each
Sheet sets – 8	$60.00 each
Pillows – 4	$20.00 each
Insurance	$123.00
Registration	$45.00
Oils and creams – 12	$15.00 each
Cleaning & supplies	$90.00

The stones used in hot stone therapy are immersed in water and heated using an electrical device to a certain temperature range. The heat of the stones relaxes muscles, improves circulation, and calms the nervous system.

a) What are her total start-up costs?

b) If Janelle has $3000.00 saved, does she have enough money to cover her start-up expenses? If not, suggest two ways she can adjust her spending to cover her start-up costs.

4. Kalie needs a $5000.00 loan to start her business. She has been offered the following loans by two different banks.

Option 1: interest rate of 3.50% per annum, compounded quarterly, for a 3-year term

Option 2: interest rate of 4.00% per annum, compounded annually, for a 3-year term

a) How much would Kalie need to repay with each option?

b) What would be the monthly payments for each option?

5. Brendan runs a furniture delivery service. His operating expenses are $4780.00 per month. If he charges $125.00 per delivery, how many deliveries must he make in a month to break even?

6. Sameera runs a landscaping business. Her revenue and expenses for one month are shown below.

The Sunken Garden, a feature of Butchart Gardens in Brentwood Bay, BC, was once an abandoned limestone quarry.

a) Classify each item as an expense or revenue.

Item	*Value*	*Revenue or Expense?*
Payment from customer	$500.00	
Soil	$56.00	
Payment from customer	$250.00	
Employee wages	$625.00	
Plants	$512.00	
Payment from customer	$700.00	
Gas for truck	$100.00	
Truck insurance	$130.00	
Truck repair	$300.00	
Employee wages	$512.00	
Payment from customer	$1000.00	
Telephone bill	$80.00	

b) Did Sameera's business have a profit or a loss for the month? Calculate the profit or loss.

7. Eldon has taken out a loan for $24 750.00 to buy a truck for his business. The term is 5 years and the interest rate is 3.40% per annum, compounded monthly.

   a) How much will Eldon have to pay back in total?

   b) How much interest will he pay?

   c) What will be his monthly payments?

8. Monique has leased a minivan for her home daycare. The conditions of the agreement are as follows:

   - term: 3 years
   - monthly payments: $675.00
   - security deposit: $500.00
   - delivery charge: $700.00
   - kilometre allowance: 24 000/year ($0.10/km for each additional kilometre)

   The residual value at the end of the lease term is $14 950.00.

   a) How much will it cost to lease the vehicle over a 3-year term? Assume Monique returns the vehicle in good condition, having driven 70 000 km.

   b) If Monique purchases the vehicle at the end of the lease term, how much will she have paid for it in total?

   c) If Monique buys the vehicle instead of leasing it, it will cost $29 900.00, including taxes and fees. What is the difference in price between purchasing the vehicle and leasing it, then purchasing it at the end of the lease term?

All child safety seats and booster cushions sold in Canada have an expiry or useful life date on them to ensure that owners have the most up-to-date and safest product available.

Glossary

accuracy: how close a measured value is to the true value

arithmetic mean: the sum of a set of values divided by the number of values; a measure of central tendency

break even: the point at which business expenses equal sales revenue

compound interest: interest calculated on the principal plus interest accumulated in prior compounding periods

compounding periods: the time between calculations of interest

correlation: the degree of relatedness between variables

cosine law: a relation between the sides of a triangle and the cosine of one of its angles
$a^2 = b^2 + c^2 - 2bc \cos A$

dependent variable: a variable whose value relies on the values of the other variables; represented as the *y*-value in an ordered pair

dilation: the enlargement or reduction of a shape or object

direct linear relation: a linear relationship that includes the ordered pair (0, 0)

experimental probability: a ratio of the number of occurrences of a particular event during an experiment or trial to the total number of trials conducted

fixed expenses: expenses that occur on a regular basis and do not change in value

independent variable: a variable whose value may be freely chosen; represented as the *x*-value in an ordered pair

lease: an agreement to pay for the use of a vehicle for a specific period of time; similar to renting

line of best fit: a line drawn on a scatterplot to show the general trend in the data

linear relation: a relationship between two variables that forms a straight line when you plot values of ordered pairs on a graph

loss: the amount by which expenses exceed sales for a given period

measure of central tendency: a measure of the midpoint of an ordered data set

median: the midpoint of an ordered data set—half the values are higher and half are lower; a measure of central tendency

mode: the value that occurs most often in a set of data; a measure of central tendency

nominal value: the target value for a measurement

oblique triangle: a triangle with no angle equal to 90°

odds: a ratio that compares a desired event to the number of undesired events, or vice versa

operating expenses: the ongoing cost of space, equipment, supplies, and other items required to run a business

origin: the intersection of the *x*- and *y*-axes of a coordinate plane; identified by the coordinates (0, 0)

outlier: a data point that lies significantly outside the general trend of the data

partial linear relation: a linear relation that does not include the ordered pair (0, 0)

percentile: a number from 1 to 99 that shows what percentage of values in a data set are lower than the given value

percentile rank: the percentage of values in a data set that are lower than a given value

polygon: a closed figure of three or more sides lying on a single plane

precision: the smallest unit of measurement on the measuring device being used (for example, millimetres on a metre stick)

principal: the original amount of money borrowed

probability: the chance of an event happening, expressed as a percentage, fraction, decimal, or in words

profit: the amount by which revenue exceeds expenses for a given period

quadrilateral: a four-sided polygon lying on a single plane

reflection: a mirror image created by transforming an original image over a reflection line

reflection line: a line across which a figure is reflected

regular polygon: a polygon with congruent side lengths and congruent interior angles

residual value: the vehicle's value when the lease is over, usually specified in the lease agreement

revenue: amount earned through the sale of goods or services

rotation: the movement of a shape or an object around a fixed point

scale factor: a number by which all the dimensions of an original figure are multiplied to produce an enlargement or reduction

scatterplot: a graph of ordered pairs, used to evaluate what type of relationship (if any) exists between two variables

similar figures: figures that are the same shape; their corresponding angles must be congruent and their corresponding sides must be proportional

sine law: for all triangles, the ratio of an angle to the side opposite is equal to the ratio for all angle/opposite side pairs

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

slope: in a linear relation, the ratio of the vertical distance over the horizontal distance; the formula is:

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

start-up costs: the goods and services you may need to purchase before you begin operating your business

supplementary angles: two angles that add up to 180°

term: the length of time over which money is borrowed

theoretical probability: the ratio of one or more events of interest to the total number of possible outcomes

tolerance: the measurement variation allowed so that an item is acceptable for its intended purpose

transformation: the movement of a shape or an object by changing its position, size, and/or shape

translation: the horizontal and/or vertical movement of a shape or an object

transversal: a line that intersects two or more lines at two distinct points

triangle: a polygon consisting of three line segments and three angles

trimmed mean: the mean calculated by disregarding an equal number of data points from both ends of the range

uncertainty: the margin of error of a measurement; if not stated otherwise, uncertainty is half the precision of the measuring device used

variable expenses: expenses that change in value or frequency

weighted mean: the mean calculated when certain data entries are worth more than others

***y*-intercept:** the *y*-value of the point where the graph of a relation intersects the *y*-axis

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Appendix

Conversion tables

LENGTH

Système International (SI)

The following measurements are equivalent:

millimetre (mm)	1000
centimetre (cm)	100
decimetre (dm)	10
metre (m)	1
decametre (dam)	0.1
hectometre (hm)	0.01
kilometre (km)	0.001

Imperial

1 ft	12 in
1 yd	3 ft
1 mi	1760 yd

SI to imperial

1 mm	0.0394 in
1 cm	0.394 in
1 m	1.094 yd
1 km	0.621 mi

Imperial to SI

1 in	2.54 cm
1 ft	0.305 m
1 yd	0.915 m
1 mi	1.61 km

MASS

SI

1 kg	1000 g
1 tonne	1000 kg

Imperial

1 lb	16 oz
1 ton	2240 lb

SI to imperial

1 kg	2.20 lb
1 tonne	0.984 ton

Imperial to SI

1 lb	0.454 kg
1 ton	1.02 tonne

TEMPERATURE

SI to imperial

$$F = \frac{9}{5}C + 32$$

Imperial to SI

$$C = \frac{5}{9}(F - 32)$$

VOLUME

SI

1 litre	1000 mL
1 mL	1 cm^3

Imperial

1 pint	16 oz
1 qt	2 pints
1 gal (US)	4 qt

SI to imperial

1 mL	0.0352 oz
1 L	0.264 gal (US)

Imperial to SI

1 oz	28.4 mL
1 pint	0.568 L
1 qt	0.946 L
1 gal (US)	3.79 L

Reference sheet

Linear Relations

Slope:

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Direct linear relation: $y = mx$

Partial linear relation: $y = mx + b$

Tolerance

Ways of expressing manufacturing tolerances:

maximum value
minimum value

$$\text{nominal value} \pm \frac{1}{2}(\text{tolerance})$$

$$\text{minimum value}^{+\text{tolerance}}_{-0}$$

$$\text{maximum value}^{+0}_{-\text{tolerance}}$$

Statistics

Mean:

$$\bar{x} = \frac{\text{sum of values}}{\text{number of values}}$$

Percentile ranking:

$$PR = \frac{b}{n} \times 100$$

Probability

Probability:

$$P(A) = \frac{\text{number of occurrences of event } A}{\text{total number of possible outcomes}}$$

Odds

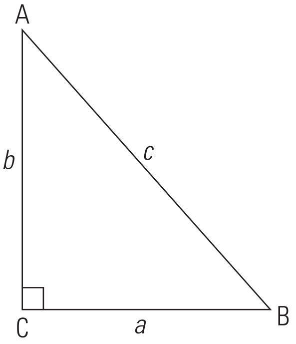
odds in favour =
favourable outcomes:unfavourable outcomes

odds against =
unfavourable outcomes:favourable outcomes

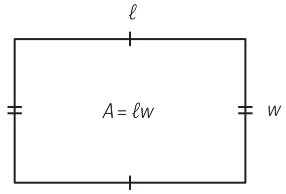
Geometry

Pythagorean theorem:

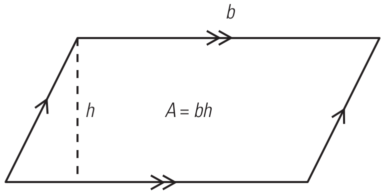
$a^2 + b^2 = c^2$



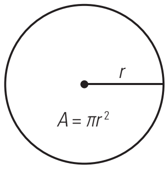
Area of geometric figures:



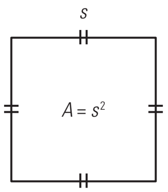
Rectangle



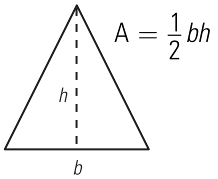
Parallelogram



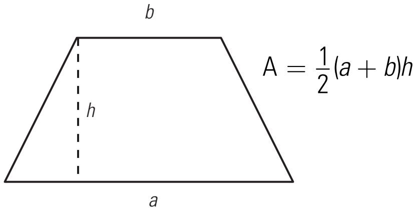
Circle



Square



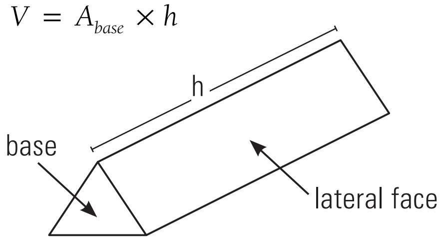
Triangle



Trapezoid

Volume of prism:

$V = A_{base} \times h$



Polygons

sum of interior angles = 180°(n – 2)

Trigonometry

Primary trigonometric ratios:

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Sine law:

$$\frac{\sin \mathrm{A}}{a} = \frac{\sin \mathrm{B}}{b} = \frac{\sin \mathrm{C}}{c}$$

Cosine law:

$$a^2 = b^2 + c^2 - 2bc \cos \mathrm{A} \quad \text{or}$$

$$\cos \mathrm{A} = \frac{b^2 + c^2 - a^2}{2bc}$$

Compound interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where:

A is the amount of money you have to repay for the loan.

P the principal.

r is the annual interest rate as a decimal.

n is the number of compounding periods per year.

t is the term of the loan in years.

Graph paper

Notes

Notes